MDL Development Team
Currently Active Developers, and their roles, principle expertise in developing MDL

- Nick Holford (University of Auckland; Uppsala University) - MDL lead, pharmacometric concepts, pharmacology, modelling and simulation tasks, use case development, NONMEM, NM-TRAN to MDL conversion.
- Mike K Smith (Pfizer) - MDL co-lead, documentation, ddmore R package lead, interoperability, model-based drug development, Pharma industry perspective, statistical concepts, use case development, grammar.
- Stuart Moodie (Eight Pillars Ltd / Pfizer) - Language design and implementation, MDL-IDE development, conversion to PharmML
- Maciej Swat (EMBL-EBI) - PharmML lead
- Florent Yvon (EMBL-EBI) - libPharmML lead
- Emmanuelle Comets (INSERM) - Design Object, Optimal Design task specification.
- Paolo Magni (Università degli Studi di Pavia) - Prior Object definition, Bayesian Use Cases, BUGS lead.
- Elisa Borella, Letizia Carrara, Cristiana Larizza, Lorenzo Pasotti, (Università degli Studi di Pavia) - BUGS conversion, BUGS connector, Bayesian Use Cases.
- Zinnia Parra-Guilén (Universidad de Navarra; Freie Universität Berlin) - Training material, documentation, Use Case specification and testing.
- Niklas Hartung (Freie Universität Berlin) - Use Case specification, testing, Monolix.
- Niels Rode-Kristensen (Novo Nordisk) - Interoperability lead
- Richard Kaye (Mango Solutions) - Integration Framework technical lead, SEE builds
- Gareth Smith (Cyprotec) - Common converter
- Henrik Bjugård Nyberg (Mango Solutions / Uppsala University) - PharmML to NM-TRAN translation
- Eric Blaudez (Lixoft) - PharmML to MLXTRAN conversion.
- Nadia Terranova (Merck Serono S.A., a Subsidiary of Merck KGaA) - Standard Output object definition
- Rikard Nordgren (Uppsala University) - NONMEM to Standard Output object translation, PopED conversion.
- Kajsa Harling (Uppsala University) - PsN translation and integration
- Chris Muselle (Mango Solutions) - ddmore R package developer

MDL Contributors
These individuals have made important contributions to MDL and associated deliverables. In alphabetical order:

- Roberto Bizzotto (CNR Institute of Neurosciences)
- Letizia Carrara (Uni di Pavia)
- Phylinda Chan (Pfizer)
- Marylore Chenel (Servier)
- Ronald Gieschke (Roche)
- Lutz Harnisch (Pfizer)
- Andrew Hooker (Uppsala University)
- Mats Karlsson (Uppsala University)
- Charlotte Kloft (Freie Universität Berlin)
- Natallia Kokash (Leiden University; UCL, London)
- Marc Lavielle (INRIA)
- Giulia Lestini (INSERM, Paris)
- Andrea Mari (CNR Institute of Neurosciences)
- France Mentré (INSERM)
- Mateusz Rogalski (Mango Solutions)
• Maria-Luisa Sardu (Merck Serono S.A., a Subsidiary of Merck KGaA)
• Gunnar Yngman (Uppsala University)
Changes have been made to the MDL since the Public Beta release on 11 Dec 2015. The changes are reflected in this documentation by striking out comments that are not applicable any more, and by adding new syntax in red, boxed-out text, as seen here.

The primary focus of MDL in this release is translation to valid PharmML, rather than conversion to target software. The previous release was primarily concerned with demonstrating interoperability across key software targets. In this version of MDL there may be MDL features which are not supported in conversion from PharmML to certain target software, but which are valid for model description and which generate valid PharmML. The aim is to widen the scope of models which can be encoded in MDL and generate PharmML for uploading to the DDMoRe repository and for future interoperability. Translation of these models to target software will follow with updates to the interoperability framework converters.

The changes to MDL since draft 7 (v0.7) enable integration of the Prior and Design Objects and improved validation of MDL giving increased confidence in generation of valid PharmML. In order to facilitate this, certain changes to syntax have been made that are NOT backwards compatible. This is regrettable since it means that existing models required changes. We do not make these changes lightly.

Key changes which break backwards compatibility:

- **DECLARED_VARIABLES** block must now have type assigned to variables to enable validation of variable types between MDL objects, particularly Design Object variables.

- Correlations and covariances between parameters are now specified in the Model Object and must be named parameters. This is to facilitate specification of priors on these parameters.

- In the Parameter Object, user should not specify the type of variability definition (**type is sd**, **type is var**, **type is corr**, **type is cov**) for VARIABILITY parameters. The variability, covariance or correlation type is specified and used in the RANDOM_VARIABLE_DEFINITION block where these parameters are defined.

- Left hand side transformations for INDIVIDUAL_PARAMETERS are no longer valid. These were felt to be confusing.

- Right hand side functions for INDIVIDUAL_PARAMETER and OBSERVATION definitions are now list definitions with the matching type to the function. This, combined with conditional statements allows more flexibility in parameter and observation definitions.

- Non-continuous outcomes (binary, count, categorical) must be defined as RANDOM_VARIABLE_DEFINITION(level=DV){ ... } and then the variable defined assigned as an anonymous
list in the OBSERVATION block. This is to ensure that outcome variables are always defined with variability at the DV level. (This is implicit in continuous outcomes due to definition of residual error at the DV level).

- Arbitrary equations defining the outcome variable are not allowed in the OBSERVATION block. Use "type is userDefined" instead.

Additional features in the new version:

- Prior Object definition
- Design Object definition
- Support for ProbOnto distributions in RANDOM_VARIABLE_DEFINITION.
- Support for target software specific Task Properties object.
- Support for DATA_DERIVED_VARIABLES where dose amounts and dose times can be derived from data columns which are being used otherwise as “use is amt” or “use is idv”.
- Support for vectors and matrices
- Support for conditional assignment to lists.
- Support for definition of parameters in RANDOM_VARIABLE_DEFINITION i.e. definition of CL - Normal(mean=POP_CL, sd=PPV_CL) for subsequent use in INDIVIDUAL_PARAMETERS. This combined with support for ProbOnto definitions allows the user to define multivariate distributions of parameters, mixture distributions etc.
- Support for combination of compartment definitions with differential equations.
- Support for model input variables passed from the Data Object with “use is variable”. This equates to “regressor” type inputs to models.
- Support for userDefined specification of the relationship between model predictions and residual error random variables in the OBSERVATION block.
# Contents

1  Introduction                                                                                             10  
   1.1  Why write a new language?                                                                           10  
   1.2  Integrated language standards                                                                     11  
   1.3  MDL in use                                                                                          11  
   1.4  MDL components and structure                                                                     12  
      1.4.1  Independence of MDL objects                                                                   13  
   1.5  Task Execution with the ddmore R package                                                          14  
   1.6  The MDL Integrated Development Environment                                                        15  
   1.7  On interoperability                                                                               15  
   1.8  Evolution of MDL                                                                                    15  

2  The Data Object                                                                                         17  
   2.1  Subsetting data for analysis                                                                      17  
   2.2  DATA_INPUT_VARIABLES Block                                                                       18  
      2.2.1  Defining dose amount                                                                        19  
      2.2.2  Defining the independent variable                                                          20  
      2.2.3  Defining the dependent variable                                                            21  
      2.2.4  Mapping data variables to model variability levels                                         22  
      2.2.5  Defining covariates                                                                       24  
      2.2.6  Defining model inputs or time-varying covariates                                           25  
      2.2.7  Assignment to a single variable using “variable = <NAME>”                                25  
      2.2.8  Assignment to multiple variables using “define = …”                                         25  
   2.3  SOURCE Block                                                                                       26  
   2.4  DECLARED_VARIABLES Block                                                                         26  
   2.5  DATA_DERIVED_VARIABLES Block                                                                     27  

3  The Parameter Object                                                                                   28  
   3.1  STRUCTURAL Block                                                                                   28  
      3.1.1  Note on parameter values                                                                      28  
   3.2  VARIABILITY Block                                                                                 28  
      3.2.1  Parameter naming                                                                            30  
      3.2.2  Covariances and Correlations                                                                 30  

4  The Model Object                                                                                       32  
   4.1  On interoperability                                                                               32  
   4.2  IDV Block                                                                                         33  
   4.3  COVARIATES Block                                                                                 33  
   4.4  STRUCTURAL_PARAMETERS Block                                                                    34  

6
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.4</td>
<td>Non-parametric and empirical distributions as priors - inline data.</td>
<td>66</td>
</tr>
<tr>
<td>6.4.1</td>
<td>Non-parametric distribution specification with inline data.</td>
<td>66</td>
</tr>
<tr>
<td>6.4.2</td>
<td>Empirical distribution specification with inline data.</td>
<td>66</td>
</tr>
<tr>
<td>6.5</td>
<td>NON_CANONICAL_DISTRIBUTION block</td>
<td>68</td>
</tr>
<tr>
<td>6.5.1</td>
<td>PRIOR_SOURCE block</td>
<td>68</td>
</tr>
<tr>
<td>6.5.2</td>
<td>INPUT_PRIOR_DATA block</td>
<td>68</td>
</tr>
<tr>
<td>7</td>
<td>The Design Object</td>
<td>70</td>
</tr>
<tr>
<td>7.1</td>
<td>Design Object overview</td>
<td>70</td>
</tr>
<tr>
<td>7.2</td>
<td>DECLARED_VARIABLES block</td>
<td>70</td>
</tr>
<tr>
<td>7.3</td>
<td>INTERVENTION block</td>
<td>70</td>
</tr>
<tr>
<td>7.4</td>
<td>SAMPLING block</td>
<td>72</td>
</tr>
<tr>
<td>7.5</td>
<td>POPULATION block</td>
<td>73</td>
</tr>
<tr>
<td>7.6</td>
<td>STUDY_DESIGN block</td>
<td>75</td>
</tr>
<tr>
<td>7.7</td>
<td>DESIGN_SPACES block</td>
<td>77</td>
</tr>
<tr>
<td>7.8</td>
<td>DESIGN_PARAMETERS block</td>
<td>78</td>
</tr>
<tr>
<td>8</td>
<td>The MOG Object</td>
<td>80</td>
</tr>
<tr>
<td>8.1</td>
<td>INFO Block</td>
<td>80</td>
</tr>
<tr>
<td>8.2</td>
<td>OBJECTS Block</td>
<td>80</td>
</tr>
<tr>
<td>8.3</td>
<td>Mapping of variable names between MDL Objects</td>
<td>81</td>
</tr>
<tr>
<td>9</td>
<td>MDL Language Reference</td>
<td>82</td>
</tr>
<tr>
<td>9.1</td>
<td>Core syntactic elements</td>
<td>82</td>
</tr>
<tr>
<td>9.1.1</td>
<td>Keywords</td>
<td>82</td>
</tr>
<tr>
<td>9.1.2</td>
<td>Variable names</td>
<td>82</td>
</tr>
<tr>
<td>9.1.3</td>
<td>Literals</td>
<td>82</td>
</tr>
<tr>
<td>9.1.4</td>
<td>Expressions</td>
<td>83</td>
</tr>
<tr>
<td>9.1.5</td>
<td>Attributes, Arguments, Properties and Values</td>
<td>91</td>
</tr>
<tr>
<td>9.1.6</td>
<td>Statements</td>
<td>92</td>
</tr>
<tr>
<td>9.1.7</td>
<td>Blocks</td>
<td>95</td>
</tr>
<tr>
<td>9.1.8</td>
<td>Objects</td>
<td>95</td>
</tr>
<tr>
<td>9.2</td>
<td>The Type System</td>
<td>96</td>
</tr>
<tr>
<td>9.2.1</td>
<td>The types</td>
<td>96</td>
</tr>
<tr>
<td>9.2.2</td>
<td>The default type</td>
<td>97</td>
</tr>
<tr>
<td>9.2.3</td>
<td>Type promotion</td>
<td>97</td>
</tr>
<tr>
<td>9.2.4</td>
<td>Typing of more complex types</td>
<td>97</td>
</tr>
<tr>
<td>9.2.5</td>
<td>Type inference rules for Symbols</td>
<td>101</td>
</tr>
<tr>
<td>9.3</td>
<td>Scoping and statement ordering</td>
<td>101</td>
</tr>
</tbody>
</table>
1 Introduction

The Model Description Language (MDL) and the Pharmacometrics Markup Language standard (PharmML) have been developed to convey information about pharmacometrics models and tasks. The goal of each language is to do this consistently between modellers (using MDL) and between software target tools (using PharmML).

MDL is a human writeable and human readable language designed to describe pharmacometric models. It is intended to be largely agnostic about the choice of target tool. MDL should facilitate clear and unambiguous definition of models, with information conveyed in a consistent manner to the PharmML representation and onwards to the target software specific code.

An important concept in the MDL is the separation of data, parameter, model and task descriptions into independent objects rather than combining these in a single file (such as in NONMEM). This supports reuse and interchange of the objects which define each component of the model and related modelling task. This independence means that model objects stored in the DDMoRe Model Repository may be combined with user objects outside the repository e.g. a Model Object, Parameter Object and a Task Properties Object may be taken from the Repository and combined with user defined Data Object. This may be useful when a user wishes to assess whether a library model is predictive for their dataset, as a preliminary step before further model refinement.

These facets (target software agnostic code + independence of MDL objects) mean that model definition using MDL is more verbose than code written specifically for any specific target tool. However the principle concept of MDL is that model code is written once and used in many different tools. For estimation, simulation, optimal design. So time spent writing code initially is saved in the longer term since MDL eliminates the need to recode models for different tasks and different software tools.

1.1 Why write a new language?

A key deliverable of the DDMoRe project is a unified Model Description Language (MDL), based on established principles, designed to be easily read and written. It is designed to facilitate easy uptake by modellers already experienced in other model definition languages, and will allow the definition of any model-based analysis.

Several languages have been created to support M&S activities. Examples of widely used languages are NONMEM (NMTRAN), Monolix (MLXTRAN), BUGS and MATLAB. However, none are shared, creating difficulties for comparison and integration. Many tools have overlapping functionality, and so the choice of one tool over another is driven largely by user preference, availability of tools, experience of the analyst and whether there is sufficient experience readily available to the analyst to provide support and advice on model building techniques specific for the tool in question. Considerable effort is currently required when moving the model from one software tool to another, as models always have to be recoded in the target software tool language, by hand. A significant need exists to rectify this situation, which DDMoRe is addressing.

---

Another common situation is using models which were developed by a third party using software that we do not have available. In that case we must try to re-encode the model before we can start using it or developing it further. This can be difficult because we need to ensure that we have all the information to construct the model in a different language. Do we have all the necessary files, settings, subroutines, functions available to us? Are the assumptions used in the model adequately annotated or described in supporting documentation? Do we have understanding of any tool-specific “tricks” and techniques that allow the model to work in the original software?

MDL provides a user interface to describe models using a common language standard. The aim is that the user writes the model (Model Object) once, in MDL, then uses this model in the tools they require (and have available) in order to complete their M&S tasks, without any tool specific recoding. This interoperability is a core deliverable of the DDMoRe project.⁶

Additionally, MDL is intended as a standard for communication of models. An analyst who only uses one tool may wish to convey their model to a third party. MDL provides the means to describe the model in a way that is consistent and provides complete information about the model (without any reference to target tool specific code). MDL is designed to focus on describing WHAT the model conveys, rather than focussing on the HOW of implementation. This has an impact on the structure and features used in MDL, but it should aid clarity and reduce ambiguity.

1.2 Integrated language standards

As described above, MDL provides the user focused layer of model description. This facilitates user understanding and model sharing between analysts.

PharmML provides the software interchange standard within DDMoRe to facilitate the transfer of models between target tools by ensuring that all of the necessary information about the model is captured and can be translated automatically to any given target tool that has an appropriate PharmML converter.

ProbOnto⁷ is an ontology and knowledge base that has been developed to describe probability distributions in a consistent and unambiguous way, as well as defining their functions, characteristics and the relationships between distributions.

The Standard Output object (SO) standard provides a consistent format for M & S results and outputs. Its availability as an object within R provides interchange and integration between existing R packages for M & S tasks within the DDMoRe infrastructure.

MDL, PharmML and the SO are the basis for interoperability which is one of the core deliverables of the DDMoRe project.

1.3 MDL in use

Very few models can be retrieved from a repository or library, be fit to any given set of data and pronounced valid for inference without further assessment or changes. Thus, the process of fitting models to data, assessing the fit through model diagnostics is an iterative process, culminating in selecting the model which is parsimonious and fit for its purpose in the inferential step - decision making, making predictions for future populations of interest, selecting dose or dosage regimen etc. The combination of features in MDL and the “ddmore” R package facilitates that process. MDL’s structure makes changes to data, models, parameters, tasks transparent - making it clear exactly which elements are changing and which are constant across steps. Using an R script to define M & S

---


https://sites.google.com/site/probonto/
task workflow facilitates an unbroken workflow for a given model and dataset, from exploratory analysis, estimation, diagnostics and simulation across a variety of tools without having to recode the model.

1.4 MDL components and structure

The MDL objects are typically defined in a file with extension .mdl. Models may also be stored and retrieved from the DDMoRe Repository either as MDL or PharmML. The key concept in MDL is that these objects can be passed to any target software for use in modelling tasks: estimation, model diagnostics and evaluation, simulation, optimal design.

An overview of the currently specified MDL objects is shown in Figure 1.
Figure 1 MDL Objects

The MDL is used to specify the inputs and the model used in an M & S task. It does this with four MDL objects defining the model, parameters, data and task properties. An additional object is used to specify the group of objects required for a given task which is known as the Modelling Objects Group (MOG).

The Model Object is the core element of the MDL and Modelling Objects Group (MOG). It defines the mathematical and statistical properties of the model by defining the structural, covariate, variability and observation models. While other objects may change depending on task, the Model Object will typically be unchanged for tasks associated with that model e.g. data visualisation, parameter estimation, model diagnostics, prediction, simulation or optimal design.

The Data Object describes the source of the data and the attributes of each of the data variables. It allows the user to define the inputs to the model and how these inputs and observed data are to be used in definition of the model. It may also be used with data visualisation tools without a Model Object.

The Design Object defines the design parameters - interventions, sampling schedules, covariate distributions, populations, study arms - for optimal design or design evaluation, and also for simulation. The Design Object replaces the Data Object in these cases.

The Parameter Object provides values for structural and variability parameters, including bounds on the parameter values for use in estimation. These can be fixed or initial values with associated constraints for parameter estimation or an instantiation of model parameters for use in making predictions or simulations.

The Prior Object defines prior distributions and values of the parameters when performing Bayesian estimation of parameters. It replaces the Parameter Object in this case.

The Task Properties Object contains settings specific to the task which will be passed on to the target software e.g. when estimating parameters it will define the estimation algorithm and associated settings for the algorithm.

It is through combination of the Model Object with other objects that we instantiate the model - linking inputs and observations from the Data or Design Object, parameter values from the Parameter or Design Object and information about the task settings in the Task Properties Object with the Model Object to form a Modelling Objects Group (MOG) ready for executing a modelling, simulation or optimal design task.

Objects are defined and stored in a MDL file with extension .mdl. It is possible to define more than one Model, Data, Design, Parameter Prior and Task Properties Object within a single MDL file. The MOG Object defines specific individual objects within an MDL file for a given task. Most commonly there will be one object of each type of information in a MDL file used for a task.

1.4.1 Independence of MDL objects

Typically, existing software has used control files that bring the elements in MDL (data, parameters, model and task definitions) together in control, model or project file(s). What is new in the MDL is the concept that the elements of the Modelling Object Group (data, parameters, model, task properties) are distinct and independent, allowing the user to combine new data and parameters applicable to their situation with an existing model. Within a M&S task workflow it is easy to see how the core Model Object remains unchanged between estimating parameters, performing model diagnostics, making predictions and simulation future outcomes. The fact that the elements of the MDL are exchangeable also makes it easier to see exactly what elements change between these M & S workflow steps.

Independence of objects also means that the Model Object should be independent of the data and, as a consequence, more easy to read and interpret without needing to have the data to hand. Having an independent Task Properties Object means that the user may store their preferred settings for tasks and target software, suitable for reuse across models and modelling tasks, to facilitate comparison between target software and to ensure reproducibility of results.
Independence of the MDL objects entails defining the contents of each object in isolation. Variables from another MDL object must be declared in the object in which they are referred to e.g. if we need to refer to the Model Object OBSERVATION block variable Y in the Data Object DATA_INPUT_VARIABLES block then we must declare a matching variable Y in the Data Object.

1.5 Task Execution with the ddmore R package

To perform tasks with the model, the user will need to use the ddmore R package. This package contains functions for executing commonly used tasks on the MDL file. The R functions can read and parse MDL objects from a MDL file to create R object representations of MDL, which can then be manipulated within R. These representations of the MDL objects can be combined to form a MOG and then written back to a new MDL file. This means that an MDL file can contain the core Model Object and associated Data, Design, Parameter, Prior and multiple Task Properties Objects which can then be combined into MOGs ready to perform specific tasks. This aids reproducibility since with one MDL file, data set and associated R script the user can perform multiple steps in a pharmacometric workflow for a given model.

Estimation using the estimate function takes as input the user specified MDL file or a MOG object defined within R. Additional functions allow the user to call modelling tools such as Perl speaks NONMEM (PsN)\(^8\). Each task produces a Standard Output (SO) object in R which may be the final output or used in subsequent tasks using functions from the ddmore R package or other R packages and commands.

Using R as the language for defining the workflow for M & S tasks with MDL objects allows analysts to tap into existing R packages for performing those tasks. The ddmore package R functions are provided to read and extract information from the SO object and to convert between this, Xpose\(^9\), mlxR\(^10\), PFIM\(^11\) and PopED\(^12\) R packages. The ddmore R package will extend and enhance what the analyst can do with existing R packages through the common standards that the DDMoRe project brings.

Task properties and settings defined in the Task Properties Object of MDL are distinct from arguments to the R functions for executing tasks. The Task Properties Object provides information to the appropriate target software about the particular settings and options required for a given task. The R function arguments are command line settings or options which are employed when invoking the target software. The Task Properties Object may define the estimation algorithm and associated settings for NONMEM, but the command line options for PsN provided from the ddmore functions govern how NONMEM should be called by PsN. For example, Task Properties specifies an ESTIMATION block with estimation method set to FOCEI, while the arguments of the bootstrap.PsN function in R allow the user to set bootstrap options from PsN such as “threads”, “stratify_on” etc. (See PsN bootstrap documentation for more details on PsN bootstrap options).

---

\(^8\) Lindbom L, Ribbing J, Jonsson EN, Perl-speaks-NONMEM (PsN)—a Perl module for NONMEM related programming, Computer Methods and Programs in Biomedicine, Volume 75, Issue 2, August 2004, Pages 85-94


\(^10\) INRIA POPIX team. [http://simulx.webpopix.org/](http://simulx.webpopix.org/)


\(^12\) Foracchia, M, Hooker, A, Vicini, P, Ruggeri, A. poped, a software for optimal experiment design in population kinetics, *Computer Methods and Programs in Biomedicine*, 74, 2004, Pages 29-46, ISSN 0169-2607, [http://dx.doi.org/10.1016/S0169-2607(03)00073-7.](http://dx.doi.org/10.1016/S0169-2607(03)00073-7.)


14
1.6 The MDL Integrated Development Environment

The MDL Integrated Development Environment (MDL-IDE) is a software platform for writing models with MDL. The MDL editor within the MDL-IDE implements the “rules” of the language through recognising MDL constructs and having a defined grammar and it ensures that MDL models are syntactically correct and result in valid PharmML. The MDL-IDE also provides additional tools - giving access to an R editor and console - so that the user can not only develop models, but execute tasks with them and define task workflow through R scripts.

The MDL-IDE gives warnings when the user writes MDL that will result in valid PharmML, but where MDL constructs are used that may not be interoperable. It gives errors when the user writes code that breaks MDL grammar rules and that will result in invalid PharmML.

1.7 On interoperability

A key goal of the DDMoRe project is to have an interoperability framework in which models are written in a consistent language, translated to PharmML and from there converted to target software code. Before the DDMoRe project no existing language standard existed across target software used in pharmacometrics modelling, and while the underlying models could be expressed consistently in mathematical and statistical terms, the implementation of any given model varied by tool and by user according to their experience with a given target software tool.

There is some flexibility within MDL around how the user can express the mathematical and statistical models. Having flexibility allows the user to encode models quickly in a common language (MDL) which can then be shared with others and mutually understood. This flexibility also facilitates encoding in a given target when that language construct does not have a parallel in other tools. However, we STRONGLY encourage the user to encode the majority of models in a way that will facilitate interoperability. There are MDL constructs that facilitate interoperability - these generally appear as built-in functions which translate to specific constructs in PharmML and the target software. These constructs cover many typical models and are designed to allow the user to generate code quickly and have high confidence that it will be interoperable across tools.

The Model Description Language Interactive Development Environment (MDL-IDE) should assist the user in ensuring that the models encoded are valid MDL (and as a consequence, also valid PharmML). Not all models will result in code which can be readily converted to all target tools.

These interoperability constructs will be highlighted in the subsequent sections, but users should pay particular attention to sections on the use of GROUP_VARIABLES, INDIVIDUAL_VARIABLES and the MODEL_PREDICTION.

1.8 Evolution of MDL

Development of MDL has been led and influenced by domain experts in M & S, computer language development, system interchange language development (markup languages), and developers of software systems. In developing MDL we have looked at features in established M&S languages, as mentioned above, and aimed to pick out features that will facilitate interoperability, while retaining the flexibility in these languages to describe complex models. The current MDL implementation focusses on interoperability in order to demonstrate that capability. The language standards in MDL, PharmML and SO are the key to eliminating the recoding necessary to pass models between tools used for different M&S tasks.

The MDL language attempts to balance consistency and clarity in definitions, with interoperability and flexibility in translation to PharmML and on to target software. It will continue to evolve to incorporate new features, extending the range of models that can be expressed using the language.

Trying to define a language that maps to all possible models as defined in all possible tools is virtually impossible. However, having a well-defined software interchange standard (PharmML) and mapping MDL into PharmML allows us to focus on describing model features with one target in mind - PharmML. The two languages - MDL and PharmML - have evolved during the course of the project. The aim is that...
these two languages should go “hand in hand” – that MDL should convey in an accessible, user (analyst) friendly way, the models that can be encoded in PharmML.

Converter tools then interpret the PharmML rather than the MDL for each software target. Testing this conversion and comparing output downstream allows us to check that the translation results in comparable models.

Future pharmacometrics tools could provide converters to import and export PharmML or use MDL directly as the model specification language. It is our hope that the DDMoRe standards would facilitate more consistency, better understanding of models as well as interoperability between modelling and simulation tools in the future.
2 The Data Object

The Data Object defines the attributes of the source data file and defines how values in the data source are to be used in the context of a given Model Object. The Data Object is independent of other blocks, including the Model Object. It must ultimately provide appropriate information to the Model Object, but as with other Objects in MDL it must be self-contained, so variables from other Objects which are referenced must be declared in the DECLARED_VARIABLES block.

The following blocks are defined within the Data Object: DECLARED_VARIABLES, DATA_INPUT_VARIABLES, SOURCE, DATA_DERIVED_VARIABLES, FUNCTIONS.

2.1 Subsetting data for analysis

For quality assurance and audit purposes it is imperative that there be a clear and traceable path between the original data source and data used in analysis. This is normally achieved in one of two ways: through having data manipulation steps performed in a scriptable language to create the dataset for use in analysis, or through having the original data as the input to the analysis and using filtering and subsetting commands in the target analysis software code to define what records are used in the task.

The “NONMEM” method for dealing with outliers and filtering data - commenting out data rows using a specific character as the first item on a data line or specifying conditions under which data is accepted or ignored - is not supported within MDL.

We suggest that users use scriptable languages like R to subset, filter and manipulate data prior to analysis, write the data for analysis to file and then modify the MDL Data Object to reference the appropriate source file in the SOURCE block. This aids reproducibility since the data used in estimation will be equivalent across target software tools. If the “ignored” or “dropped” data is saved separately in a file or listing then it will also be possible to quickly and easily verify that the data used in analysis, when combined with the dropped data matches the original dataset.

For legacy data and NMTRAN models, we recommend using the “ignored” function within the “metrumrg” R package (https://r-forge.r-project.org/projects/metrumrg/). This function reads a NONMEM control stream and creates a TRUE / FALSE logical flag for whether the records meet IGNORE and/or ACCEPT criteria specified in the NMTRAN code. This will allow the user to identify which data records have been dropped by NONMEM. Filtering on the original data based on this criteria will allow them to create a dataset ready for analysis with MDL.

For example, if the NMTRAN control file had the following $DATA statement:

$DATA mx2007.csv IGNORE=&
    IGNORE ID.EQ.1
    ACCEPT VISI.EQ.3

This code means that any data rows which start with “@” are to be omitted, that the subject with ID == 1 should be omitted from analysis, and only VISI == 3 is to be included.

Using the metrumrg function ignored, we have the following code which can be used within the R script. This assumes that the $DATA statements above are in a control file called “run1.ctl”:

library(metrumrg)
mx2007 <- read.csv("mx2007.csv", header=T)
mx.dropped <- mx2007[ignored(ctlfile="run1.ctl"),]
mx.kept <- mx2007[!ignored(ctlfile="run1.ctl"),]
This separates out the dropped records into the mx.dropped data frame, and the kept records into the
data frame mx.kept. The user can then write mx.kept to a .csv file and use this as the file in the
SOURCE block.

2.2  DATA_INPUT_VARIABLES Block

This block defines the columns in the dataset described in SOURCE block and how these map to model
variables.

- All columns of the data file defined in the SOURCE block must be defined in DATA_INPUT_VARIABLES. This aids clarity and readability.
- In the current MDL, variables in the DATA_INPUT_VARIABLES block must be defined in the
column order they appear in the SOURCE block data file.
- In the current MDL, variable names in the DATA_INPUT_VARIABLES block must match the
names in the header row of the data file named in the SOURCE block. For interoperability
with Monolix the case of the names in the header row must match the case of the data
variable in the DATA_INPUT_VARIABLES block.
- DATA_INPUT_VARIABLES cannot have more than one use defined.
- All DATA_INPUT_VARIABLES must have a use defined.
- Columns in the data which are not required in the model should have “use is ignore”.

The typical syntax for defining items in the DATA_INPUT_VARIABLES block is:

<Variable name> : { use is < use type > }

Define types for use type are:

<table>
<thead>
<tr>
<th>Use</th>
<th>Defines</th>
</tr>
</thead>
<tbody>
<tr>
<td>id</td>
<td>Individual identifier. Typically subject ID in clinical trials. Defines the</td>
</tr>
<tr>
<td></td>
<td>individual level of parameter variability.</td>
</tr>
<tr>
<td>idv</td>
<td>Independent variable. Typically TIME. In the Model Object, the reserved</td>
</tr>
<tr>
<td></td>
<td>variable T is used as the integrator variable in the specification of differential equations.</td>
</tr>
<tr>
<td>amt</td>
<td>Dose amount</td>
</tr>
<tr>
<td>dv</td>
<td>Dependent variable</td>
</tr>
<tr>
<td>varLevel</td>
<td>Defines a level of variability in the model. Not required for DATA_INPUT_VARIABLES with use is id or use is dv. Used to define all other variability levels such as interoccasion variability.</td>
</tr>
<tr>
<td>covariate</td>
<td>Covariate for use in definition of fixed effect variable</td>
</tr>
<tr>
<td>catCov</td>
<td>Categorical covariate for use in definition of fixed effect variables based on categories</td>
</tr>
<tr>
<td>variable</td>
<td>Defines any model input not covered by other types. Corresponds to the “regressor” variable type in Monolix.</td>
</tr>
<tr>
<td>dvid</td>
<td>Dependent variable identifier when there is more than one type of observation.</td>
</tr>
<tr>
<td>evid</td>
<td>Event identifier. This type is defined in MDL, but not yet implemented by converters in the current SEE.</td>
</tr>
<tr>
<td>mdv</td>
<td>Missing dependent variable</td>
</tr>
</tbody>
</table>
2.2.1 Defining dose amount

The dosing amount is defined through DATA_INPUT_VARIABLES with “use is amt”. It is assumed that when the column has a non-zero / non-missing value, this amount is assigned to the relevant Model Object dosing variable.

The syntax for defining the dose amount is:

```
<Variable> : { use is amt, variable = <mdlObject variable> }
```

For example:

```
AMT : { use is amt, variable = GUT }
```

For the current version of the interoperability framework SEE, the name of this variable must be AMT or DOSE.
Within the Model Object, the dose amount is defined when AMT > 0 if the \texttt{DATA\_INPUT\_VARIABLES} variable is defined as “\texttt{use is amt}”. It is not necessary to conditionally assign a Model Object variable to this value when AMT > 0. This is taken care of in translation to PharmML behind the scenes. If, however, the user chooses to treat the dose amount column as “\texttt{use is covariate}” or “\texttt{use is variable}” then this \textit{will} need conditional assignment within the model. Also, in that case the dosing amount variable should be declared in the \texttt{COVARIATES} block of the Model Object.

For analytical models (such as UseCase2) the dosing amount may be defined with respect to a dosing variable in the Model Object, rather than as an initial amount in a differential equation or compartment. In that case it is necessary to declare the dosing variable within the \texttt{MODEL\_PREDICTION} block:

```
The declared dosing variable for analytical models should have type ::dosingTarget. This maps into variables of type ::dosingVar and ::dosingTarget in the Model Object. Examples of ::dosingTarget are COMPARTMENT variables with “type is depot”, “type is direct” and also variables defined by differential equations.
```

For example:

In the Data Object:

```
\texttt{DECLARED\_VARIABLES[ D::dosingTarget ]}
\texttt{DATA\_INPUT\_VARIABLES{ AMT : { use is amt, variable=D } }}
```

In the Model Object:

```
\texttt{MODEL\_PREDICTION { D :: dosingVar # dosing variable } k = CL/V }  
\hspace{1cm} \texttt{CC = if ( T < TLAG) then 0}
\hspace{1cm} \hspace{1cm} \texttt{else (D/V) * KA/(KA-k) * (exp(-k * (T - TLAG))- exp(-KA*(T-TLAG)) )} 
\texttt{}} # end MODEL\_PREDICTION
```

MDL supports definition of multiple doses via \texttt{DATA\_INPUT\_VARIABLES} defined as “\texttt{use is ii}”, “\texttt{use is addl}”, “\texttt{use is ss}”.

MDL supports infusion rate (zero-order input rate) specification via \texttt{DATA\_INPUT\_VARIABLES} with “\texttt{use is rate}”. The current version of MDL does not support negative values for “\texttt{use is rate}” in order to allow model defined rate or duration.

Please also read the section 2.2.7 and 2.2.8 on assignment using “\texttt{variable = ...}” compared to using “\texttt{define = ...}”.

### 2.2.2  Defining the independent variable

The independent variable is defined through a \texttt{DATA\_INPUT\_VARIABLES} with “\texttt{use is idv}”. The Model Object has an \texttt{IDV} block where the independent variable in the model is defined and the model variable defined in this block is automatically mapped to the Data Object variable defined as “\texttt{use is idv}”. This is used to link the model independent variable in the Model Object \texttt{IDV} block and the event (observation, dosing) times specified in the \texttt{DATA\_INPUT\_VARIABLES}.

Syntax:

```
\langle Variable\rangle : \{ use is idv \}
```
Typically the independent variable will be time. UseCase11 shows a pharmacodynamic model with the plasma concentration as the independent variable.

For the current version of the SEE, the name of the independent variable must be TIME.

2.2.3 Defining the dependent variable

The dependent variable is defined through a `DATA_INPUT_VARIABLES` with “use is dv”. This data variable is the observation and must be mapped to the Model Object `OBSERVATION` prediction variable using the Data Object `DECLARED_VARIABLES` block.

For the current version of the interoperability framework SEE, the name of the dependent variable must be DV.

2.2.3.1 Continuous data, single Model Object `OBSERVATION` prediction

If there is only one observation type and it is continuous then the user should map the `DATA_INPUT_VARIABLES “use is dv”` variable to a prediction variable name in the Model Object `OBSERVATION` block. It is possible to map the dependent variable to a single, specified Model Object `OBSERVATION` prediction variable using “variable = <NAME>”.

The Model Object `OBSERVATION` block prediction variable name must be declared in the Data Object `DECLARED_VARIABLES` block.

The observation variable in the `DECLARED_VARIABLES` block must have type ::observation.

For example:

In the Data Object:

```
DECLARED_VARIABLES{ Y::observation }
DATA_INPUT_VARIABLES{
...
DV : { use is dv, variable = Y }
...}
```

In the Model Object:

```
OBSERVATION{
  Y : { type is additiveError, additive=SD_ADD, eps=EPS_Y, prediction= CONC}
}# end OBSERVATION
```

2.2.3.2 Multiple Model Object `OBSERVATION` predictions

If mapping the single dataset dependent variable column (with “use is dv”) to multiple Model Object `OBSERVATION` prediction variables, it is necessary to also define a `DATA_INPUT_VARIABLE` with “use is dvid” that identifies which records of the dataset belong to each observation type. The user must also define how to map the Model Object `OBSERVATION` block variables to values in the `DATA_INPUT_VARIABLE` with “use is dvid”.

The syntax is as follows define={<value> in <data column name with “use is dvid”> as <DECLARED_VARIABLE> variable>, etc.}.

For example (UseCase3):

In the Data Object:

```
DECLARED_VARIABLES{ CP_obs::observation PCA_obs::observation }
```
DVID : { use is dvid }
DV : { use is dv, define={1 in DVID as CP_obs, 2 in DVID as PCA_obs} }

In the Model Object:

\[
\text{OBSERVATION}\
\begin{align*}
\text{CP}_\text{obs} &: \{ \text{type is combinedError1,} \\
& \quad \text{additive} = \text{RUV}_\text{ADD}, \text{proportional} = \text{RUV}_\text{PROP}, \\
& \quad \text{eps} = \text{EPS}_\text{CP}, \text{prediction} = \text{CC} \} \\
\text{PCA}_\text{obs} &: \{ \text{type is additiveError,} \\
& \quad \text{additive} = \text{RUV}_\text{FX}, \\
& \quad \text{eps} = \text{EPS}_\text{PCA}, \text{prediction} = \text{PCA} \} \\
\end{align*}
\]

This means when the data variable with “use is dvid” has the value 1 then the observation in the data variable with “use is dv” *within the same data record* will be mapped to the Model Object OBSERVATION block variable CP_obs, and when this variable has the value 2 it will be mapped to PCA_obs.

All continuous observation variables must be declared in the DECLARED_VARIABLES block as ::observation.

When declaring an outcome variable that is binary (UseCase12) or categorical (UseCase13), it is necessary in the DECLARED_VARIABLES block to define the category name for the Model Object variables which define the prediction.

For example (UseCase12):

In Data Object:

\[
\text{DECLARED_VARIABLES}\{ \text{Y withCategories}{\text{none, event}} \}
\]

Using this convention, the type is implicit in the “withCategories” keyword.

Note that when declaring an outcome variable with a Poisson count or Binomial number of successes outcome, Y is declared as the variable attribute of an anonymous OBSERVATION block list.

For example (UseCase11):

In the Data Object:

\[
\text{DECLARED_VARIABLES}\{ \text{Y::observation} \}
\]

In the Model Object:

\[
\text{RANDOM_VARIABLE_DEFINITION}(\text{level}=\text{DV})\{
\quad \text{Y} \sim \text{Poisson1}(\text{rate}=\text{LAMBDA})
\}
\]

\[
\text{OBSERVATION}\{
\quad \text{::} \{ \text{type is count, variable} = \text{Y} \}
\}
\]

2.2.4 Mapping data variables to model variability levels

The hierarchy of model levels of variability is defined within the Model Object in the VARIABILITY_LEVELS block. Within the Data Object we match the levels in the model to DATA_INPUT_VARIABLES to identify the data variables where changing values signify new individuals,
occasions or observations (and other levels of variability in the model). By default the lowest level of the hierarchy is the observation level with DATA_INPUT_VARIABLES defined as “use is dv”. The other variability level commonly used is the experimental unit, in clinical trials this is typically the subject with DATA_INPUT_VARIABLES defined as “use is id”.

The level in the hierarchy is defined by a numerical level value in the Model Object VARIABILITY LEVELS block.

For example (UseCase1):

In Data Object:

```r
DATA_INPUT_VARIABLES{
    ID : { use is id }
    ...
    DV : { use is dv, variable = Y }
    ...
}# end DATA_INPUT_VARIABLES
```

In the Model Object:

```r
VARIABILITY_LEVELS{
    ID : { level=2, type is parameter }
    DV : { level=1, type is observation }
}
```

Other variability levels may be defined in DATA_INPUT_VARIABLES as “use is varLevel”. This will be used to define variability levels such as occasion, study (if modelling across more than one study) etc.

For example, when defining occasions for use in between occasion variability models:

```r
OCC : { use is varLevel }
```

For example (UseCase8):

In Data Object:

```r
DATA_INPUT_VARIABLES{
    ID : { use is id }
    TIME : { use is idv }
    WT : { use is covariate }
    AGE : { use is covariate }
    SEX : { use is catCov withCategories {female when 1, male when 0} }
    AMT : { use is amt, variable = INPUT_KA}
    OCC : { use is varLevel }
    DV : { use is dv, variable = Y }
    MDV : { use is mdv }
}# end DATA_INPUT_VARIABLES
```

In Model Object:

```r
VARIABILITY_LEVELS{
    ID : { level=3, type is parameter }
    OCC : { level=2, type is parameter }
    DV : { level=1, type is observation }
}
```
MDL does not place any limits on the number of levels of variability within the Model Object. However, some constraints may exist within the target software used for estimation.

Note that occasionally, if modelling summary level data in a model-based meta-analysis, the experimental unit defined in the data may be different, for example, the treatment arm rather than an individual subject.

For example:

In Data Object:
```plaintext
DATA_INPUT_VARIABLES{
    ARM : { use is id }
    DV : { use is dv, variable = Y }
}# end DATA_INPUT_VARIABLES
```

In Model Object:
```plaintext
VARIABILITY_LEVELS{
    ARM : { level=2, type  is parameter }
    DV : { level=1, type  is observation }
}
```

See section 4.6 for discussion of identifying the reference level of variability when using a model with a Design Object.

2.2.5 Defining covariates

2.2.5.1 Defining continuous covariates

Continuous covariates (including time-varying covariates) are defined as “use is covariate”.

Note the discussion in sections 4.3 and 4.9 about the requirements of covariates that are to be used in definition of INDIVIDUAL_PARAMETERS.

Interpolation can be specified for continuous covariates through the argument “interp = &constInterp | &cubicInterp | &lastValueInterp | &linearInterp | &nearestInterp | &pchipInterp | &splineInterp”.

Note the “&” in front of the interpolation type to signify that a function is being referenced.

Interpolation functions take as arguments t0 (start of time interval), t1 (end of time interval), x (variable for interpolation), x0 (variable value at t0) and x1 (variable value at t1).

2.2.5.2 Defining categorical covariates

Categorical covariates are defined as “use is catCov” and must have a mapping between the values in the data column and categories to be used in the model. This serves a dual purpose: firstly, providing clarity on how numeric codes in the data map to named categories and secondly, transparency in model description by allowing us to use those named category labels in the model when referring to the categories.

The mapping is performed by using the keyword “withCategories” and then specifying the mapping between categories and values using <category> when <value> in a comma separated list.

```
SEX : {use is catCov withCategories {female when 1, male when 0} }
```

The categories defined (female, male) must match those defined within the Model Object COVARIATES block. Note that the category labels (female, male) are not character strings. They are enumerated.
variables, and can be referred to in the model as SEX.female or SEX.male. For example to define an action dependent on the SEX being female in the Model Object we would use the Boolean comparison SEX == SEX.female. This evaluates to true when the value in the SEX column matches the value defined (in the DATA_INPUT_VARIABLES as above) that corresponds to the female category of the SEX variable.

In the current version of MDL the categories defined must be unique i.e. it is not possible to assign more than one value to a category, nor is it possible to define several categories with the same name. This means that each category maps to only one data value. The following is code NOT valid:

```
food : {use is catCov withCategories {fed when 2, fasted when 1, fasted when -999}
Nor is:
food : {use is catCov withCategories {fed when 2, fasted when [-999,1] }
```

The user should identify all categories in the DATA_INPUT_VARIABLE block:

```
food : {use is catCov withCategories {fed when 2, fasted when 1, missing when -999}
```

2.2.6 Defining model inputs or time-varying covariates

Often we will want to pass data variables to the GROUP_VARIABLES or MODEL_PREDICTION blocks in the Model Object which will not fit the definition of covariates as defined in section 2.2.5. This might be the case if we want to pass individualised predictions of PK parameters into a PD model, or a time-varying covariate such as plasma concentration or age for use in a maturation model. These variables are sometimes referred to as regressors or model input variables.

These variables are defined in MDL as “use is variable”.

Variables with “use is variable” should not be used with definition of INDIVIDUAL_PARAMETERS with “type is linear”.

2.2.7 Assignment to a single variable using “variable = <NAME>”

If the value of the variable within the data is to be mapped to a single model variable e.g. dosing amount D, then the variable attribute must be assigned, and the associated variable declared in DECLARED_VARIABLES:

```
DECLARED_VARIABLES{ D::dosingTarget Y::observation }
DATA_INPUT_VARIABLES{
...
AMT : { use is amt, variable = D }
DV : { use is dv, variable = Y }
...
}
```

2.2.8 Assignment to multiple variables using “define = < ... >”

In the case of mapping data values to multiple variables depending on the values of another variable, the syntax is as follows define={<value> in <data variable name> as <declared_variable>, etc.}. The Model Object variables used in this definition must also be declared in the DECLARED_VARIABLES block.

```
DECLARED_VARIABLES{ CP_obs::observation PCA_obs::observation }
DATA_INPUT_VARIABLES{
...
2.3 SOURCE Block

This block defines the source data file for use with the model. It defines the file name and file format.

In the current version of MDL it is assumed that the SOURCE data file will be present as an ASCII comma-delimited text file (.csv). We also assume that the dataset conforms to NONMEM data standards. The data file should have a header row with names matching those in the DATA_INPUT_VARIABLES block. Data values should be numeric. Missing values should be denoted by ".".

The MDL syntax is as follows:

```mdl
<source object name> : {
file = <filename>,
inputFormat is nonmemFormat
}
```

For example:

```mdl
SOURCE {
srcfile : {file = "warfarin_conc.csv",
inputFormat is nonmemFormat }
} # end SOURCE
```

For the current version of the interoperability framework SEE, data files must be in the same folder and workspace as the model file.

2.4 DECLARED_VARIABLES Block

This block links variables defined in the Model Object with variables defined within the Data Object - occasionally we need to refer to Model Object variables while describing the data constructs. For example: When defining the Pharmacokinetic model (UseCase1) we need to define which Model Object variable receives the dosing amount - in this case the differential equation specifying the amount in the GUT, and an observation variable Y.

Since the MDL objects are independent of each other (i.e. the Data Object is not “aware” of Model Object variables) we must explicitly declare Model Object variables within the Data Object if we need to refer to them.

When declaring the variable, the user must also define the variable type. A controlled vocabulary of types is provided through the MDL-IDE. Type is defined via the double colon:

```mdl
<Variable name> :: <type>
```

For example: GUT :: dosingTarget Y :: observation.

This improves validation of the MOG since we can check that Data Object DATA_INPUT_VARIABLE use is mapped to an appropriate type of variable and that this passes through to the appropriate type of variable in the Model Object. This change is also required to ensure that the Data Object and the Design Object handle definition and declaration of variables in an equivalent way.

The following types will be typically of use in DECLARED_VARIABLES:

dosingTarget, observation.
As has been described above, when declaring an outcome variable that is binary (UseCase12) or
categorical (UseCase13), it is necessary in the DECLARED_VARIABLES block to define the categories for
the outcome variable.

Note that when declaring an outcome variable with a Poisson count or Binomial number of successes
outcome, `Y` is declared as observation.

Note that no delimiter is required between variables defined in the DECLARED_VARIABLES block.

In the current MDL, variable names mapped across MDL Objects must match e.g. if we declare
variable `Y` in the Data Object, then this will be linked to the Model Object OBSERVATION block
variable `Y`.

### 2.5 DATA_DERIVED_VARIABLES Block

Occasionally, the user will need to define new variables that depend on existing information in the
dataset for example the dose amount or dose time when this is to be used as a covariate in the model.
The DATA_DERIVED_VARIABLES block allows the user to define new variables using data in the columns
with “use is `amt`” and “use is `idv`”. The variables defined must have unique names, different from
those specified in the DATA_INPUT_VARIABLES.

Syntax is

```
<variable> : {use is <doseTime / covariate / variable / doseInterval>, ... }
```

The subsequent arguments of the list depend on its use.

```
<variable> : {use is doseTime, idvColumn = <DATA_INPUT_VARIABLE variable with “use is idv”>,
               dosingVar = <dosing variable defined with type ::dosingTarget>}
```

In the current MDL, “use is doseInterval” below cannot currently be mapped to PharmML.

```
<variable> : {use is doseInterval, idvColumn = <DATA_INPUT_VARIABLE variable with “use is idv”>,
               dosingVar = <dosing variable defined with type ::dosingTarget> }
```

```
<variable> : {use is covariate, column = <DATA_INPUT_VARIABLE variable> }
```

```
<variable> : {use is catCov, column = <DATA_INPUT_VARIABLE variable> }
```

```
<variable> : {use is variable, column = <DATA_INPUT_VARIABLE variable> }
```

For example (UseCase2_1):

```
DATA_DERIVED_VARIABLES{
  # Like 'use is `amt`' we assume that the DT variable is only assigned when AMT > 0.
  # The typing ensured that the attributes reference a column with the correct 'use'.
  DT : { use is doseTime, idvColumn=TIME, amtColumn=AMT }
}
```
3 The Parameter Object

The Parameter Object defines model parameter values for use with the Model Object. In estimation tasks, these are typically the initial values for the estimation algorithm, or fixed parameter values within the model.

The Parameter Object should provide a value for each parameter listed in the Model Object STRUCTURAL_PARAMETERS and VARIABILITY_PARAMETERS blocks.

STRUCTURAL and VARIABILITY parameter blocks are kept separate to allow the user to quickly identify the function of each parameter in the model and to facilitate certain tasks, for example fixing variability parameters for simulation.

3.1 STRUCTURAL Block

The STRUCTURAL block defines the numerical values of the structural parameters with optional constraints (low and high values) and whether the value is fixed or to be estimated. Each structural parameter must have the value argument assigned a numeric value.

For each structural parameter the typical construct will be

<PARAMETER NAME> : { value = <numeric> }

Or, with additional optional attributes

<PARAMETER NAME> : { value = <numeric>, lo = <numeric (lower bound)>, hi = <numeric (upper bound)>, fix = <true | false> }

This provides a numerical value for a parameter which may be used as an initial estimate for estimation or as a value for simulation. Numerical values may be expressed in scientific notation.

The lo and hi attributes are optional and are used to define lower and upper boundaries for estimation.

The fix attribute is optional. It may be set to a logical value of true or false. The default value of fix is false. When fix is true the parameter will not be estimated in an estimation task. Specifying “fix = true” overrides any setting of lo and hi.

```plaintext
STRUCTURAL {
    POP_CL : { value = 0.1, lo = 0.001 }
    POP_V : { value = 8, lo = 0.001 }
    POP_KA : { value = 0.362, lo = 0.001 }
    POP_TLAG : { value=1, lo=0.001 }
    BETA_CL_WT : { value = 0.75, fix = true }
    BETA_V_WT : { value = 1, fix = true }
    RUV_PROP : { value = 0.1, lo = 0 }
    RUV_ADD : { value = 0.1, lo = 0 }
}
```

3.1.1 Note on parameter values

It is typical to specify log-Normal distributions for parameters, but the user should be aware that in some models, parameters may be negative. As with other languages, the user should be careful to avoid parameterisations that would lead to taking logs of a negative number.

3.2 VARIABILITY Block

The VARIABILITY block defines the names and values of random effect parameters (including covariance or correlation parameters) that are to be used in the Model Object. Similar to the
STRUCTURAL block above, the VARIABILITY block provides initial values for estimation. Each variable must have the value argument assigned a numeric value.

The VARIABILITY block has a more complex structure because it needs to express both values for variance parameters, but also any correlations or covariances between these variance parameters (random effects).

Currently, MDL supports definition of the VARIABILITY (random effect) parameters and definition of the covariance or correlation between these parameters (see section 3.2.1).

Similar to the STRUCTURAL block, the VARIABILITY block requires attributes for each random effect used in the model.

For each random effect parameter the typical construct is

```plaintext
<PARAMETER NAME> : { value = <numeric>, type is <sd|var> }
```

With additional fix attribute

```plaintext
<PARAMETER NAME> : { value = <numeric>, type is <sd|var>, fix = true }
```

The type argument specifies whether the initial values and parameter estimation are specified on the standard deviation scale. The parameter value type must correspond to the type used in the Model Object RANDOM_VARIABLE_DEFINITION block.

Within the Parameter Object VARIABILITY block, we no longer need to specify the type of variability (variance or sd). The Parameter Object simply defines values for the parameters used in the Model Object RANDOM_VARIABLE_DEFINITION block. The user then needs to ensure that the parameter values are on the appropriate scale.

Note: that in the version of PsN used in the current version of the SEE, bootstrap estimates of variability parameters are not available on the standard deviation scale. Returned variability parameters from bootstrap estimation will be on the variance scale.

An example VARIABILITY block:

```plaintext
VARIABILITY {
    PPV.CL : { value = 0.1, type is sd }
    PPV.V : { value = 0.1, type is sd }
    CORR_CL_V : { value = 0.01 }
    PPV.KA : { value = 0.1, type is sd }
    PPV.TLAG : { value = 0.1, type is sd, fix=true }
    RUV_PROP : { value = 0.1, lo = 0 }
    RUV_ADD : { value = 0.1, lo = 0.0001 }
}
```

Note that parameter estimates for residual errors e.g. RUV_PROP and RUV_ADD above are now specified as VARIABILITY parameters, and not STRUCTURAL. Typically, these parameters are defined as multipliers of standard Normal(0,1) random variables in definition of the residual error model. The purpose of the VARIABILITY block definition in the Parameter Object is to make it easier to identify those parameters associated with variability and if required “turn off” variability by fixing these parameters to zero. Placing the residual error parameters in the STRUCTURAL parameter block made this difficult, since these parameters can have arbitrary names. Nevertheless, the models are equivalent whether residual errors are defined by parameters specified in the STRUCTURAL or in the VARIABILITY block.

Note also that correlations between parameters are given parameter values in the VARIABILITY block, but definition of correlation and covariance now occurs in the Model Object RANDOM_VARIABLE_DEFINITION block.
When defining values for multivariate distributions, the user may need to define vectors and matrices to define the mean and covariance matrix (respectively). To do so the user defines a list with the following syntax:

To define a vector of length k:

\[ \text{< VARIABLE NAME > : \{vectorValue = [ <value1>, <value2>, \ldots, <valuek> ]} \]

The square brackets denote that the result is a vector.

To define a matrix of size n rows by p columns:

\[ \text{< VARIABLE NAME > = \{ [ <value_{1_1}, <value_{1_2}, \ldots, <value_{1_p};
\text{<value_{2_1}, <value_{2_2}, \ldots, <value_{2_p};
\text{...};
\text{<value_{n_1}, <value_{n_2}, \ldots, <value_{n_p}] \}}\]

Note the double square brackets to define the matrix type, comma separated values to signify individual elements and semi-colon to specify the end of a row.

Alternatively to create a matrix, it is possible to use functions “diagonal”, “triangle”, “matrix”. These take a vector as input and return a matrix.

For example (UseCase6_2.mdl):

```
VARIABILITY

PPV_CL_V_KA : \{matrixValue = triangle([0.1,
0.01, 0.1,
0.01, 0.01, 0.1], 3, true)}

... # end VARIABILITY
```

### 3.2.1 Parameter naming

Unlike some target software, MDL does not have reserved names for parameters, nor is any meaning extracted from parameter names.

In the MDL documentation, we have used the convention that variability parameters describing the population parameter variability from the combination of between subject and within subject (between occasion) random effects are named PPV_. The individual level random effects we've named ETA_ since this is a familiar convention for many analysts. The residual unexplained variability parameters have been named RUV_ and the random variable associated with these has been named EPS_ again to following a familiar convention.

### 3.2.2 Covariances and Correlations

Random variability parameters and any covariances or correlations are defined separately, rather than as a combined matrix.

The covariance (or correlation) between random effects is defined as follows:

\[ \text{<PARAMETER NAME> : \{ parameter = \{vector of random effect variables\},
value = \{vector of values\},
type is \{cov | corr\} } \]
The random effects variables must be declared in the _DECLARED_VARIABLES_ block within the Parameter Object so they can be mapped to the random effect variables in the Model Object.

**Note that value expects a vector, so even if a single correlation is specified (one value) then this must be enclosed in square brackets to signify this is a vector with one element.**

So for a simple example where the between subject variance parameters for CL, V and KA are on the standard deviation scale and the correlation between these parameters is to be specified, the standard deviation–correlation matrix (standard deviation on the diagonal, correlation off-diagonal) is given by

\[
\begin{pmatrix}
ppv_{\text{CL}} & & \\
ppv_{\text{V}} & & \\
ppv_{\text{KA}} & & \\
\end{pmatrix} =
\begin{bmatrix}
sd = 0.1 & 0.01 & 0.01 \\
\text{corr} = 0.01 & sd = 0.1 & 0.01 \\
\text{corr} = 0.01 & \text{corr} = 0.01 & sd = 0.1 \\
\end{bmatrix}
\]

And the corresponding MDL code is:

```mdl
warfarin_PK_CORR_par = parObj {
    DECLARED_VARIABLES { ETA_CL ETA_V ETA_KA }
    STRUCTURAL { 
        _____
        _____) # end-STRUCTURAL
    VARIABILITY { 
        _____ PPV_CL : { value = 0.1, fix=true, type is sd }
        _____ PPV_V : { value = 0.1, type is sd }
        _____ PPV_KA : { value = 0.1, type is sd }
        _____ PPV_TLAG : { value = 0.1, type is sd, fix=true } 
        # correlation between CL, V, KA
        _____ OMEGA1 : { type is corr, parameter=[ ETA_CL, ETA_V, ETA_KA ], 
                         value=[0.01, 0.01, 0.01] }
        _____) # end-VARIABILITY
    ) # end of parameter object
}
```

In the code above, the variable OMEGA1 is defined as the lower triangle of the matrix above (correlation entries only) and three values are required to define the correlations between the parameters. Specifying the between subject variability parameters separately from covariances and correlations allows the user to change the covariance or correlation structure independently of the other variance parameter definitions.

If any VARIABILITY block variable associated with the correlation or covariance list definition has the attribute fix=true, then all diagonal and off-diagonal elements of the standard deviation–correlation matrix will be assumed to have the attribute fix=true.

**Note that the parameters correlated are the random effects rather than the parameters defining the distribution of the random effects. Thus it is these random effect variables that are declared in the DECLARED_VARIABLES block.**

In the current version of MDL, if a covariance or correlation is specified between three or more parameters then all elements of that covariance or correlation must be estimated.
4 The Model Object

The Model Object within the MDL is intended to describe the mathematical and statistical properties of the model. MDL defines language elements that allow the user to code a wide variety of models and in a variety of ways. The Model Object is intended to specify the model independent of the target software which will be used for the task (estimation or simulation). The same model should be able to be used for a variety of tasks - estimation, simulation or optimal design without recoding. The Model Object should also be independent of the data - where possible we use enumerated types for categorical covariates and outcomes so that definition of the model is clear to any user regardless of the data used in a given task.

It should be noted however that MDL does not guide the user about whether the model that is defined is suitable for a given purpose or for any target software. The user is free to define any model, however they must also be aware that the specified model may not be useable with all target software.

As stated in the Introduction, the Model Object is intended to convey the mathematical and statistical definitions required to completely define the model. MDL used in defining the model is intended more as a descriptive language rather than a programmatic one. The Model Object is used in tasks by combining it with Data, Parameter and Task Properties objects, and defining tasks within the R script.

Currently defined blocks are IDV, COVARIATES, POPULATION, FUNCTION, VARIABILITY_LEVELS, STRUCTURAL_PARAMETERS, VARIABILITY_PARAMETERS, GROUP_VARIABLES, RANDOM_VARIABLE_DEFINITION, INDIVIDUAL_VARIABLES , MODEL_PREDICTION, DEQ, COMPARTMENT, OBSERVATION.

Which blocks within the Model Object are used for a particular model depends on the structure of that model. Blocks should not be left empty (although this is not a syntax error). It is good practice to structure and write the model to facilitate readability and understanding of the model. Simple statements that are clear and unambiguous are preferred to statements combining many actions into one line of code. Use of the MODEL_PREDICTION block is encouraged to make it clear what the final prediction is from the model prior to use in generation of the observation level.

In the current version of MDL, the variable names and parameter names in the STRUCTURAL_PARAMETERS and VARIABILITY_PARAMETERS blocks of the Model Object must be matched to those in the Data and Parameter objects. The MOG Object brings together the Data, Parameter, Model and Task Properties Objects to perform tasks, and at this stage it is assumed that the variable names match across objects.

The independence of the Data Object from the model means that the data referenced by the Data Object may be easily used with a different model without modification of the Data Object. Similarly, the independence of the Parameter Object from the model means that all the parameters related to modelling project e.g. describing a particular drug, may be stored in one place. Note that parameters defined in the Parameter Object must have unique names.

Unlike other MDL Objects, the Model Object does not use a DECLARED_VARIABLES block. Instead variables are declared when they are used within the IDV, COVARIATES, STRUCTURAL_PARAMETERS, VARIABILITY_PARAMETERS blocks. Particular care may be required for models defined using analytic equations (rather than via differential equations, compartments). In these models it may be necessary to declare inputs such as DOSE within the MODEL_PREDICTION block.

4.1 On interoperability

The primary focus of MDL in this release is translation to valid PharmML, rather than conversion to target software. The previous Public Release was primarily concerned with demonstrating interoperability across key software targets. In this version of MDL there may be features supported which are not supported by certain target software, but which are valid for model description and which generate valid PharmML. The aim is to widen the scope of models which can be encoded in MDL.
and generate PharmML, since the latter is required for uploading models to the DDMoRe repository. Translation of these models to target software will follow with updates to the interoperability framework converters.

The MDL-IDE should assist the user in ensuring that the models encoded are valid MDL (and as a consequence, also valid PharmML).

Models in MDL may be expressed in a number of ways, which may be influenced by a number of factors including which languages the user is familiar with for encoding models. Flexibility allows the user to encode models quickly in a common language (MDL) which can then be shared with others and mutually understood. This flexibility also facilitates encoding in a given target when that language construct does not have a parallel in other tools. However, we **STRONGLY** encourage the user to encode the majority of models in a way that will facilitate interoperability. Interoperability allows the user of the model to choose the best tool for the job, or at least the tools that they have available to them.

If the user follows certain conventions for coding then it will increase the chance that a given model is interoperable between target tools. These conventions will be highlighted in the subsequent sections, but users should pay particular attention to sections 4.7, 4.9 and 4.10 on definition of GROUP_VARIABLES defining fixed effects, INDIVIDUAL_VARIABLES defining the relationship between covariates (or GROUP_VARIABLES defined variables) and random effects and MODEL_PREDICTION using these parameters to calculate predictions for given inputs.

### 4.2 IDV Block

The **IDV** block defines the independent variable within the model. Typically this is TIME (or T for differential equations). An **IDV** block must be present in the Model Object. The default for the independent variable is “T”.

The syntax is a simple variable declaration:

```
IDV{<Independent variable name>}
```

Note that the independent variable defined in the model will be mapped to the Data Object DATA_INPUT_VARIABLES variable with “use is idv”.

### 4.3 COVARIATES Block

The **COVARIATES** block declares and defines covariates to be used in the GROUP_VARIABLES, INDIVIDUAL_VARIABLES and MODEL_PREDICTION blocks (see discussion of regressors below for use of covariates in the MODEL_PREDICTION block). Covariates listed in the **COVARIATES** block must be specified as “use is covariate” or “use is catCov” in the DATA_INPUT_VARIABLES block in the Data Object or defined in the POPULATION block of the Design Object. Covariate transformations may be specified within this block.

```
COVARIATES{
    <Covariate name>
    <Categorical covariate name> withCategories {<category1>, <category2>, ..., <category_k>}
    <Covariate name> = <simple transformation equation>
}
```

For categorical covariates, the categories defined in the COVARIATES block must match those specified for DATA_INPUT_VARIABLES with “use is catCov” - see the definition of the SEX covariate above.

An example **COVARIATES** block is shown below:

```
COVARIATES{
    WT
    ...
```


SEX withCategories {female, male}
logtWT = ln(WT/70)
}

In this example the withCategories prefix to the list of category names will be used to link to the values associated with these names in the Data Object.

The logtWT variable in the COVARIATES block may be used as the value for the cov attribute in the linear function in the INDIVIDUAL_VARIABLES block if WT follows the above rules for covariates.

Please also read about the specification of covariate models in sections 4.7 and 4.9.

The definition of covariates above assumes that the covariates are constant within individuals or vary only at occasion levels. It is also possible to define covariates that vary with the independent variable (typically time):

COVARIATES (type is idvDependent) {
  WT
  logtWT = ln(WT/70)
}

Covariates which are defined as idvDependent should NOT be used in the “type is linear” definition of INDIVIDUAL_PARAMETERS.

4.4 STRUCTURAL_PARAMETERS Block
This block declares fixed effect parameters that define the structure of the model. There is no separator character in between variable names. The variable names do not need to be on separate lines, but it may be easier to read if they are presented in this way and it allows comments to be added to help communication

STRUCTURAL_PARAMETERS {
  <Variable name(s) of structural parameters>
}

For example:

STRUCTURAL_PARAMETERS {
  POP_CL
  POP_V
  POP_KA
  POP_TLAG
  BETA_CL_WT
  BETA_V_WT
  RUV_PROP
  RUV_ADD
} # end STRUCTURAL_PARAMETERS

4.5 VARIABILITY_PARAMETERS Block
Similar to the STRUCTURAL_PARAMETERS block, this block declares all the variability (including covariance, correlation and residual error) parameters (population parameter variability and other variability level parameters) used in the model. The variable names do not need to be on separate lines, but it may be easier to read if they are presented in this way and allows comments to be added to help communication

VARIABILITY_PARAMETERS {
  <Variable name(s) of variability parameters>
4.5.1 Residual Unexplained Variability

Residual variability is typically defined as a standard Normal distribution \( \sim \text{N}(\text{mean}=0, \text{var}=1) \). The standard residual error models (see section 4.12.1) then define the parameters of that model e.g. additive and proportional which multiply the random \( \text{N}(0,1) \) variable. They can also define expressions involving parameters which define the residual error model. These parameters should be declared as \text{VARIABILITY\_PARAMETERS}. As such these additive and proportional parameters are typically defined in the \text{STRUCTURAL\_PARAMETERS} block.

4.6 \text{VARIABILITY\_LEVELS} Block

The \text{VARIABILITY\_LEVELS} block defines the model hierarchy. Each variable should have attributes defining its \textit{level} in the model hierarchy and variability \textit{type} which is one of “\text{parameter}” or “\text{observation}”. \text{DATA\_INPUT\_VARIABLES} with “\textit{use is dv}” and “\textit{use is id}” are automatically identified as describing variability levels. Additional variables can be used to define variability levels by defining these as “\textit{use is varLevel}” in \text{DATA\_INPUT\_VARIABLES}.

We assume that level = 1 is the level of each observation. Additional levels of the hierarchy are added to this. Typically in population models there is at least one additional level of variability – that of the individual (the experimental unit). Occasionally if modelling summary level data in a model-based meta-analysis, treatment arm may be used as the experimental unit and labelled in the \text{DATA\_INPUT\_VARIABLES} block as “\textit{use is id}”.

\text{VARIABILITY\_LEVELS}\{
  <\text{Variable name}> : \{ \text{level} = <\text{number}>,
  \text{type} \text{ is} <\text{parameter} / \text{observation}>\}
\}

For example:

\text{VARIABILITY\_LEVELS}\{
  ID : \{ \text{level}=2, \text{type} \text{ is} \text{parameter} \}
  DV : \{ \text{level}=1, \text{type} \text{ is} \text{observation} \}
\}

If between occasion variability is required in the model then this should be specified here as a variability level between the observation and individual levels. In NONMEM occasion is typically specified as an additional layer of inter-individual variability which is defined conditionally on an occasion variable in the dataset. In MDL this is explicitly treated as a distinct level of variability.

\text{VARIABILITY\_LEVELS}\{
  ID : \{ \text{level}=3, \text{type} \text{ is} \text{parameter} \}
  OCC : \{ \text{level}=2, \text{type} \text{ is} \text{parameter} \}
\}
Additional levels of variability are easily implemented by incrementing “level = <number>” with an associated DATA_INPUT_VARIABLE with “use is varLevel”. This facilitates definition of levels such as between trial random variability.

The distinction between “type is observation” and “type is parameter” will be used further in future versions of MDL to describe models where there are additional levels of hierarchy above the individual e.g. when describing differences between trials as a random effect, or modelling population level differences; and at the observation level e.g. replicates of PD measurements at each time point, or multiple assays of a single sample.

When estimating model parameters using observed data, the DATA_INPUT_VARIABLE with “use is id” is automatically taken as the “reference level” of the model hierarchy. When using a Design Object, the reference level of the model hierarchy is likely to be implicit (subjects in a study arm) rather than explicit. In this case it will be necessary to use the following syntax:

```
VARIABILITY_LEVELS(reference=ID){
   ID : { level=3, type is parameter }
   OCC : { level=2, type is parameter }
   DV : { level=1, type is observation }
}
```

Since the Model Object is intended to be unchanging across tasks, it is good practice to include this (reference = <reference variability level variable>) syntax in all cases. In most pharmacometrics models this reference level will be the individual.

### 4.7 GROUP_VARIABLES Block

The GROUP_VARIABLES block can be used to specify group specific variables using parameters and fixed effect relationships between parameters and covariates. The INDIVIDUAL_VARIABLES block can then use these values in definition of the individual parameters by incorporating the random between individual variabilities defined in the RANDOM_VARIABLE_DEFINITION block(s).

Using the GROUP_VARIABLES block to define covariate relationships is not supported for parameter estimation in some target software since the equations defined in the GROUP_VARIABLES block are user defined. The MDL-IDE is not equipped to determine whether the defined relationships conform to linear relationships (after transformation) that have been shown to allow interoperability between software.

For this reason we suggest that definition of covariate dependent GROUP_VARIABLES is used only in cases where a reformulation to “linear” or “linear after transformation” relationships with covariates as defined in section 4.9 is not possible.

The GROUP_VARIABLES block is essential for defining relationships between structural parameters and covariates which are non-linear, even after transformation. For example to describe clearance across both adults and children a maturation model may be required. For example:

```
GROUP_VARIABLES{
   FSIZE = (WT/70) ^ 0.75
}
```

---

13 PharmML Specification 0.2.1 section 11.4 Modelling Steps, item 1.2.30
[http://docs.google.com/viewer?a=v&pid=sites&srcid=ZGVmYXVsdGRvbWFpbnxwaGFybW1sdGVtCHxneD03YzY1WNkzGQxMmQwNDk5](http://docs.google.com/viewer?a=v&pid=sites&srcid=ZGVmYXVsdGRvbWFpbnxwaGFybW1sdGVtCHxneD03YzY1WNkzGQxMmQwNDk5)
FAGE = if(AGE >= 20) then exp(BETA_CL_AGE*(AGE-20)) else 1
FMAT = 1/(1+(PCA/TM50)^(-HILL))
GRP_CL = POP_CL * FSIZE * FAGE * FMAT

GRP_CL can then be used in the definition of individual variables within the INDIVIDUAL_VARIABLES block.

4.7.1 Defining model constants
Model constants (model variables with constant values) may be defined in MDL within the GROUP_VARIABLES block.

However, to ensure interoperability within the current SEE, constant values in the model should be defined as STRUCTURAL_PARAMETERS and fixed to a value in the Parameter Object.

For models expressed as systems of differential equations (DEQ block), model variables can be set to constant values in the MODEL_PREDICTION block, but this may be computationally inefficient in the target software implementation.

4.8 RANDOM_VARIABLE_DEFINITION Block
The RANDOM_VARIABLE_DEFINITION block defines the distribution of the random effects to be used in construction of mixed effects models. The RANDOM_VARIABLE_DEFINITION block defines random variables in terms of parametric distributions.

It is assumed that all variables within the same block are defined for the same level of the model hierarchy. Separate RANDOM_VARIABLE_DEFINITION blocks should be used for each layer of the model hierarchy. The user specifies which level through the "(level = <name of variable associated with this level>)" syntax following the RANDOM_VARIABLE_DEFINITION block name.

The following syntax is used to define random variables

```
RANDOM_VARIABLE_DEFINITION(level = <VARIABILITY_LEVEL variable>) {
  <VARIABLE NAME> ~ <Distribution with arguments>
}
```

The RANDOM_VARIABLE_DEFINITION block supports probability distributions as specified in the ProbOnto\textsuperscript{14} knowledge base\textsuperscript{15}. Typically for definition of structural parameter and residual error random variability, Normal distributions will be used. The MDL distribution "Normal(...)" maps to either ProbOnto Normal1 or Normal2 depending on the parameterisation:

<table>
<thead>
<tr>
<th>MDL Name</th>
<th>Argument name</th>
<th>Argument Types</th>
<th>ProbOnto distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>mean</td>
<td>Real</td>
<td>Normal1</td>
</tr>
<tr>
<td></td>
<td>sd</td>
<td>Real</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{14} Swat, MJ; Grenon, P; Wimalaratne, S. ProbOnto: ontology and knowledge base of probability distributions. Bioinformatics (2016); doi: 10.1093/bioinformatics/btw170
\textsuperscript{15} https://sites.google.com/site/probonto/download
This means that the user does not need to remember which ProbOnto distribution uses which parameterisation for this frequently used distribution.

An example of `RANDOM_VARIABLE_DEFINITION` for individual random effects is given below:

```plaintext
RANDOM_VARIABLE_DEFINITION(level=ID) {
    ETA_CL ~ Normal(mean = 0, sd = PPV_CL)
    ETA_V ~ Normal(mean = 0, sd = PPV_V)
    ETA_KA ~ Normal(mean = 0, sd = PPV_KA)
    ETA_TLAG ~ Normal(mean = 0, sd = PPV_TLAG)
} # end RANDOM_VARIABLE_DEFINITION
```

In the code above, ETA_CL, ETA_V, ETA_KA and ETA_TLAG vary with each new value of ID. These variables are normally distributed with mean = 0 and standard deviation defined by the variability parameters. The distribution can also be defined using variances. **Note that in the current MDL version, the use of standard deviation or variance must match what is specified for that parameter in the Parameters Object VARIABILITY block.**

Correlations and covariances between the ETAs are defined within the Parameter Object. This allows the user to test different correlation and covariance structures between variables without having to change the Model Object. Typically the random effect variables will be Normally distributed, so covariances and correlations between variables imply a multivariate Normal distribution.

In the example above, all random variability parameters are independent. To specify correlation or covariance between parameters, the user should specify either pairwise correlation or covariance between random variables or use a multivariate distribution. In contrast to the previous version of MDL where correlations and covariances were defined only in the Parameter Object, this version requires the user to specify correlations and covariances in the `RANDOM_VARIABLE_DEFINITION` block. This is to allow the Prior Object to define priors on parameters used in the Model Object.

The following syntax is used to define correlation or covariance:

```plaintext
:: {type is <correlation / covariance>,
    rv1 = <RANDOM_VARIABLE_DEFINITION variable>,
    rv2 = <RANDOM_VARIABLE_DEFINITION variable>,
    variable = <VARIABILITY_PARAMETERS parameter> }
```

Note the use of the "anonymous list" using double colon "::". This is used since we are assigning additional information to the variable defined in the `VARIABILITY_PARAMETERS` block.

For example (UseCase1):

```plaintext
:: {type is correlation, rv1=ETA_CL, rv2=ETA_V, value=CORR_CL_V}
```

Alternatively, the user can specify multivariate distribution(s) for parameters to specify the joint distribution of multiple random variability parameters. To do this, the user must specify the type of parameters in the `STRUCTURAL_PARAMETERS` (if required) and `VARIABILITY_PARAMETERS` blocks. Typically for multivariate distributions, there may be a vector of mean values, and a matrix of correlations or covariances. We can then use these to define the multivariate distribution using ProbOnto definitions.

So if we assume that the random effects for CL, V and KA (ETA_CL, ETA_V and ETA_KA in the univariate case) come from a multivariate distribution then the distribution of the vector ETA_CL_V_KA is given as:

```plaintext
```
\[
ETACL\_V\_KA = \begin{bmatrix} ETA\_CL \\ ETA\_V \\ ETA\_KA \end{bmatrix} ~ \text{MultivariateNormal}(\text{mean} = [0,0,0], \text{covariance} = PPV\_CL\_V\_KA)
\]

where \(PPV\_CL\_V\_KA\) is a covariance matrix defining the variances and covariances of the random effects:

\[
PPV\_CL\_V\_KA = \begin{pmatrix} PPV\_CL & COV\_CL\_V & COV\_CL\_KA \\ COV\_CL\_V & PPV\_V & COV\_V\_KA \\ COV\_CL\_KA & COV\_V\_KA & PPV\_KA \end{pmatrix}
\]

Note that the ProbOnto distribution MultivariateNormal1 uses mean and covariance, while MultivariateNormal2 uses mean and correlation.

For example (UseCase6_2):

```plaintext
VARIABILITY_PARAMETERS {
PPV\_CL\_V\_KA::matrix
PPV\_TLAG
RUV\_PROP
RUV\_ADD
} # end VARIABILITY_PARAMETERS

RANDOM_VARIABLE_DEFINITION(level=ID) {
ETA\_CL\_V\_KA ~ MultivariateNormal1(mean = [0,0,0], covarianceMatrix = PPV\_CL\_V\_KA)
ETA\_TLAG ~ Normal(mean = 0, var = PPV\_TLAG)
} # end RANDOM_VARIABLE_DEFINITION
```

Similarly for the residual unexplained variability with mean 0 and a fixed variance of 1, we might have a RANDOM_VARIABLE_DEFINITION block as follows:

```plaintext
RANDOM_VARIABLE_DEFINITION(level=DV) {
EPS\_Y ~ Normal(mean = 0, var = 1)
}
```

To define between occasion variability we might have a RANDOM_VARIABLE_DEFINITION block as follows:

```plaintext
RANDOM_VARIABLE_DEFINITION(level=OCC) {
eta\_BOV\_CL~ Normal(mean=\theta, var=BOV\_CL)
eta\_BOV\_V~ Normal(mean=\theta, var=BOV\_V)
eta\_BOV\_KA~ Normal(mean=\theta, var=BOV\_KA)
eta\_BOV\_TLAG~ Normal(mean=\theta, var=BOV\_TLAG)
} # end RANDOM_VARIABLE_DEFINITION
```

Note that in the above example blocks, ID, DV and OCC are declared as valid identifiers for the variability hierarchy through the VARIABILITY_LEVELS block assuming appropriate specification within the Data Object of DATA_INPUT_VARIABLES with “use is id”, “use is varLevel” and “use is dv” for ID, OCC and DV (respectively).

```plaintext
VARIABILITY_LEVELS{
ID : { level=3, type is parameter }
OCC : { level=2, type is parameter }
DV : { level=1, type is observation }
}
```
The `RANDOM_VARIABLE_DEFINITION` block can also be used to explicitly define the distribution of individual variables, however if this method is used, then an anonymous list must be specified in the `INDIVIDUAL_VARIABLES` block to refer to random variable defined in this way. Parameters defined in this way cannot be used with “type is linear” or “type is general” in the `INDIVIDUAL_VARIABLES` block.

For example:

```plaintext
RANDOM_VARIABLE_DEFINITION(level=ID) {
    CL ~ LogNormal3(median = POP_CL, stdevLog = SD_LNCL)
    ...
}
INDIVIDUAL_VARIABLES{
    :: {type is rv, variable=CL}
    ...
}
```

Covariates can be included via definition in the `GROUP_VARIABLES` block, but this may limit interoperability and translation of the model to target software.

For example:

See `GROUP_VARIABLES` block example above for definition of GRP_CL.

```plaintext
RANDOM_VARIABLE_DEFINITION(level=ID) {
    CL ~ LogNormal3(median = GRP_CL, stdevLog = SD_LNCL)
    ...
}
```

The `RANDOM_VARIABLE_DEFINITION` block is also used to specify non-continuous outcome variables i.e. count, binary, categorical.

The current version of MDL supports only Normal distributions for random effects.

### 4.9 INDIVIDUAL_VARIABLES Block

The `INDIVIDUAL_VARIABLES` block is used to express how the fixed effect variables (population parameters, covariates with their associated fixed effect parameters) and random effects (defined in the `RANDOM_VARIABLE_DEFINITION` block) combine to define the individual variables which will be used in the `MODEL_PREDICTION` block to calculate predictions for given inputs. If this is not a population model or if variables are completely defined through the `GROUP_VARIABLES` block then this block is not required. However, that might break interoperability with some tools like Monolix, which require the definition of individual parameters.

There are three principle ways of defining `INDIVIDUAL_VARIABLES` and these will be described below.

The only way of defining `INDIVIDUAL_VARIABLES` that is currently supported for parameter estimation across target software is the “linear after transformation” method described in section 4.9.1 below.
4.9.1 Mixed effect model with linear fixed effects and normally distributed random effects

In some cases it is possible to express the fixed effects of covariates for a population parameter as a linear model with normally distributed random effects, sometimes employing a simple transformation (log, logit etc.) to achieve this.

We refer to this as a linear covariate model and this equates to the following mathematical definition:

\[ h(\psi_i) = h(\psi_{pop}) + \beta C_i + \eta_i \]

- \( \psi_i \) - Individual parameter
- \( \psi_{pop} \) - Typical or population mean parameter
- \( \beta \) - Fixed effects
- \( C_i \) - Covariates
- \( \eta_i \) - Random effect
- \( h \) - Transformation function - typically log, logit, probit etc.

The MDL syntax for this form of specification is:

```mdl
<Individual parameter> :
    {type is linear, trans is <h>,
        pop = <Population STRUCTURAL parameter>,
        fixEff = [ {coeff = <Fixed Effect STRUCTURAL parameter for covariate>,
                   cov = <Covariate in COVARIATES block conforming to rules below>},
                   … <Additional coefficient and covariate pairs as above> ],
        ranEff = [ RANDOM_VARIABLE_DEFINITION parameter(s) ] )
```

The `fixEff` and `trans` arguments are optional.

Note that the syntax has changed from `<parameter> = linear(…)` to `<parameter> : {type is linear, …}`. This change allows conditional assignment of list types to the parameter. So if the model for a parameter varies according to a covariate or variable value, then this can be reflected in the condition applied.

Note that left hand side transformations of the individual parameters are no longer allowed. The transformation specified in the `trans` argument applies to both the left hand side and right hand side of the equation. The `ranEff` argument expects a vector of random variables. If there is only one random variable the square brackets are not required.

Note that \( h(\text{Individual parameter}) \) on the left hand side of the equation is a function whereas the \( <h> \) in “trans is <h>” is an enumerated type. These should be the same transformation. If a transformation is used, then the Individual parameter (left hand side of the equation) is implicitly back-transformed for use in later calculations, for example in the `MODEL_PREDICTION` block.

For example (UseCase1):

```mdl
CL : {type is linear, trans is ln, pop = POP_CL,
      fixEff = [{coeff=BETA_CL_WT, cov=logtWT}],
      ranEff = ETA_CL }
```
Using this construct for individual variables equates to the MU referencing approach in NONMEM and the standard definition of individual parameters in Monolix.

As discussed in the documentation of the Data Object defining covariates (section 3.3.5.1) certain constraints are placed on the type of covariate used in this form of specification. When covariates are defined within the Model Object COVARIATES block and used in the specification of INDIVIDUAL_PARAMETERS using the linear(..., fixEff= [{coeff=<coefficient>, cov = <covariate>}] ) construct, they must have particular properties:

- They must be constant within an individual or constant within an occasion.
- They will only be allowed simple transformations within the model e.g. centering on a median / mean and/or log transformation, logit transformation.
- The transformation cannot depend on another covariate.
- Statistical models (random effects) on covariates are not supported.

If a categorical covariate is used, then the catCov argument in fixEff should refer to the appropriate category of the covariate. For example (UseCase5):

```plaintext
CL : {type is linear, trans is ln,  
    pop = POP_CL,  
    fixEff = [  
        {coeff = BETA_CL_WT, cov = logtWT},  
        {coeff = POP_FCL_FEM, catCov = SEX.female },  
        {coeff = BETA_CL_AGE, cov = tAGE}  
    ],  
    ranEff = ETA_CL }
```

If the categorical covariate has more than k>2 categories then the user needs to specify k-1 dichotomous “dummy” covariates to specify the factor levels and appropriate contrasts (between level k and an appropriate comparison value). For example when adding GENOTYPE as a categorical covariate, the user may want to compare each category of GENOTYPE to a suitable reference value of the covariate. In this case all of the “dummy” dichotomous comparison covariates should be included in the model in one step.

If between occasion variability is specified in a RANDOM_VARIABLE_DEFINITION block then the associated random effects can be specified in a vector form of the ranEff attribute. These will be added into the linear equation. For example (UseCase8):

```plaintext
CL : {type is linear, trans is ln, pop = POP_CL,  
    fixEff = [{coeff=BETA_CL_WT, cov=logtWT}],  
    ranEff = [eta_BSV_CL, eta_BOV_CL ]}
```

### 4.9.2 General mixed effect model with Gaussian random effects.

The second formulation for the INDIVIDUAL_PARAMETERS block uses variables defined in the GROUP_VARIABLES block and assumes that the random effect is additive i.e. is Gaussian (Normally distributed) or Gaussian after transformation.

We refer to this as a “general or Gaussian after transformation” model and the associated mathematical representation is:

\[ h(\psi_i) = H(\beta, C_i) + \eta_i \]

- \( \psi_i \) - individual parameter
- \( \beta \) – Fixed effects

\[ 42 \]
\(Ci - \) Covariates
\(\eta_i - \) Random effect(s)
\(H - \) Arbitrary function
\(h - \) Transformation function - log, logit, probit.

Where \(H(\beta, Ci)\) is defined in the \textit{GROUP_VARIABLES} block.
The MDL syntax for this form of specification is:

\[
<\text{Individual parameter}>
: \{\text{type is general},
\begin{align*}
\text{grp} &= <\text{GROUP_VARIABLES defined variable }>, \\
\text{trans} &= \langle \text{ln / logit / probit} \rangle, \\
\text{ranEff} &= [ \text{RANDOM_VARIABLE_DEFINITION parameter(s)} ]
\end{align*}
\]

The \textit{trans} argument is optional.

Note that there is no "trans is" function argument on the right hand side. If the \textit{trans} argument is used, then it is assumed that appropriate transformations have been made in the \textit{GROUP_VARIABLES} block or in the assigned value for the \textit{grp} attribute to ensure that the fixed effect and random effect are additive and on the correct scale given the transformation.

For example, for the \textit{GROUP_VARIABLES} defined in section 4.7 above:

\[
\text{CL} : \{\text{type is general, } \text{grp} = \ln(\text{GRP_CL}), \\
\text{trans is } \ln, \\
\text{ranEff} = \text{ETA_CL}\}
\]

This corresponds to the following equation for \(CL\):

\[
\ln(CL) = \ln(\text{GRP_CL}) + \text{ETA_CL}
\]

which, after back-transformation is equivalent to:

\[
CL = \text{GRP_CL} \times \exp(\text{ETA_CL})
\]

4.9.3 \textbf{Mixed effect model defined by equations}

The individual variables can also be defined using expressions by combining parameters with variables defined in \textit{GROUP_VARIABLES} and random effects.

For example:

\[
\text{CL} = \text{POP_CL} \times \exp(\text{ETA_CL})
\]

Or (using a variable \text{GRP_CL} defined in the \textit{GROUP_VARIABLES} block as defined above)

\[
\text{CL} = \text{GRP_CL} \times \exp(\text{ETA_CL})
\]

It is also possible to define fixed and random effect expressions as follows:

\[
\text{CL} = \text{POP_CL} \times (\text{WT/70})^{0.75} \times \exp(\text{eta_PPV_CL})
\]

Which can be log transformed into a linear form of mixed effect model like this:

\[
\text{CL} = \exp(\ln(\text{POP_CL}) + 0.75 \times \ln(WT/70) + \text{eta_PPV_CL})
\]
However, note that while it is possible for the user to “see” that the equation above is linear in the fixed and random effects, it is not possible for the MDL-DE to determine this. To specify linear models we must explicitly do so using the \{\text{type is linear}, \ldots \} construct described in section 4.9.1.

### 4.9.4 \textbf{INDIVIDUAL_VARIABLES} without inter-individual variability

For interoperability reasons, parameters defined in the \textbf{STRUCTURAL_PARAMETERS} block without associated variability i.e. where the individual value is the same as the parameter value, should be defined within the \textbf{INDIVIDUAL_VARIABLES} block. If the model parameter is constrained to be positive, then an appropriate transformation should be used to ensure a positive value.

### 4.9.5 \textbf{INDIVIDUAL_VARIABLES} where the variable is defined in the \textbf{RANDOM_VARIABLE_DEFINITION} block.

As discussed in section 0 above, if the individual variable is defined completely via a distribution in the \textbf{RANDOM_VARIABLE_DEFINITION} block, then an anonymous list must be used to declare the variable within the \textbf{INDIVIDUAL_VARIABLES} block.

The syntax for the anonymous list is:

```
:: \{\text{type is rv, variable = <RANDOM_VARIABLE_DEFINITION variable>}\}
```

### 4.9.6 Conditional assignment of \textbf{INDIVIDUAL_VARIABLES}

It is possible to apply conditional handling to the assignment of \textbf{INDIVIDUAL_VARIABLES}. Note that the conditioning occurs on the RIGHT HAND SIDE of the expression ONLY. The conditioning statement should follow the conventions described in section 9.1.4.4.

The syntax is as follows:

```
<Individual variable>

: if(condition1) then \{ INDIVIDUAL_VARIABLE list 1 \}

elseif(condition2) then \{ INDIVIDUAL_VARIABLE list 2 \}

else \{ INDIVIDUAL_VARIABLE list 3 \}
```

Note the use of an “else” statement to ensure that \texttt{<Individual parameter>} is always assigned a value.

For example:

```
CL : if(RF==RF.normal) \{ \text{type is linear, trans is ln,}

\hspace*{1cm} pop = POP_CL, 
\hspace*{1cm} fixEff = \{coeff = BETA_CL_WT, cov = logtWT\},
\hspace*{1cm} ranEff = ETA_CL \}

else \{ \text{type is general,}

\hspace*{1cm} grp = ln(GRP_CL),
\hspace*{1cm} trans is ln, 
\hspace*{1cm} ranEff = ETA_CL \}
```

In the above, the expression for individual Clearance is conditional on whether the subject’s renal function (RF) is “normal” or not. The user would need to have defined a suitable model for GRP_CL in
the \texttt{GROUP\_VARIABLES} block. (RF is a categorical data variable that has a symbolic value of “normal” and other categories which are not used in this example).

Please also see sections 9.1.4.4 and 9.1.4.5 for further information on handling of conditional statements.

\textbf{4.9.7 \texttt{INDIVIDUAL\_VARIABLES} definitions in practice.}

As has been discussed above, to facilitate interoperability we strongly suggest that users try to formulate their models using the \{\texttt{type} is \texttt{linear}, ... \} form shown in section 4.9.1 with the caveat included about the forms of covariate relationships that can be used within this construct.

In some cases, users may have to consider how their model is constructed more carefully. For example, in a pharmacodynamic model:

\[
PD = PD\_BASELINE + PD\_BETA*CP + ETA\_PD
\]

It may be tempting to try to write this as a \{\texttt{type} is \texttt{linear}, ... \} relationship with \texttt{CP} as a covariate, but recall that covariates may not be time-varying, and \texttt{CP} would almost certainly break this rule.

If we encode \texttt{GRP\_PD} = \texttt{POP\_BASELINE} + \texttt{POP\_BETA*CP} as a \texttt{GROUP\_VARIABLE} and then add \texttt{ETA\_PD} in \texttt{INDIVIDUAL\_VARIABLES} using the \{\texttt{type} is \texttt{general}, ... \} form then the \texttt{GRP\_PD} is also time-varying.

\[
\text{INDIV\_PD} : \{\texttt{type} is \texttt{linear}, \ \texttt{pop} = \texttt{GRP\_PD}, \ \texttt{ranEff} = \{\texttt{ETA\_PD}\}\}
\]

However if we break the above model into components, then we can use \{\texttt{type} is \texttt{linear}, ... \} to express an individual baseline

\[
\text{INDIV\_BASE} : \{\texttt{type} is \texttt{linear}, \ \texttt{pop} = \texttt{POP\_BASELINE}, \ \texttt{ranEff} = \{\texttt{ETA\_BASE}\}\}
\]

\[
\text{INDIV\_BETA} : \{\texttt{type} is \texttt{linear}, \ \texttt{pop} = \texttt{POP\_BETA}, \ \texttt{ranEff} = \{\texttt{ETA\_BETA}\}\}
\]

We can then move the linear relationship with \texttt{CP} to the \texttt{MODEL\_PREDICTION} block

\[
\text{MODEL\_PREDICTION}\{
PD = INDIV\_BASE + INDIV\_BETA*CP
\}
\]

Using the \texttt{INDIVIDUAL\_VARIABLES} block to define individual parameters which are then used in \texttt{MODEL\_PREDICTION} should allow most models to be interoperable.

\textbf{4.10 \texttt{MODEL\_PREDICTION} Block}

The \texttt{MODEL\_PREDICTION} block is where the structural model predictions are defined. Calculations use mathematical expressions that may involve the population parameters (structural) as well as group and individual variables (parameters).

If a \texttt{MODEL\_PREDICTION} block is not supplied this is not an error but requires that any prediction referred to in the \texttt{OBSERVATION} blocks has been defined using variables in a \texttt{GROUP\_VARIABLES} or \texttt{INDIVIDUAL\_VARIABLE} block.

For example below we present the \texttt{MODEL\_PREDICTION} block using variables \texttt{DOSE}, \texttt{V}, \texttt{CL}, \texttt{V} and \texttt{TIME}.

\[
\text{MODEL\_PREDICTION}\{
\text{DOSE::dosingVar} \ # \text{recall that DOSE must be declared before use in analytical models.}
\text{CONC}=\text{DOSE}/V*\text{exp}(-\text{CL}/V*\text{TIME})
\}
\]
If a DEQ sub-block is specified then variables calculated within the DEQ sub-block can be referred to outside of this block to calculate the model prediction. An example is given below.

To ensure interoperability, any variable used in the MODEL_PREDICTION block must be either:

- the independent variable
- defined in MODEL_PREDICTION
- declared in INDIVIDUAL_VARIABLES using \{type is linear, ... \}
- defined as “use is variable” in the DATA_INPUT_VARIABLES block of the Data Object

This implies in particular that STRUCTURAL_PARAMETERS, VARIABILITY_PARAMETERS, GROUP_VARIABLES and random variables defined in RANDOM_VARIABLES_DEFINITION cannot be used in MODEL_PREDICTION.

4.10.1 DEQ Sub-block

Use of a DEQ sub-block is optional - differential equations may be used anywhere in the MODEL_PREDICTION block - but it is encouraged to use this sub-block for clarity and readability of the resulting code.

The DEQ sub-block specifies the structural model through differential equations. The general form is

\(<VARIABLE> : \{ \text{deriv} = <expression>, \text{init} = <\text{Real number}>, x0 = <\text{Real number}> \} \)

The DEQ sub-block combines equations and differential equations and the resulting system of equations is integrated across the independent variable, usually time.

\( \text{init} = <\text{Real number}> \) is the initial value of the differential equation

\( x0 = <\text{Real number}> \) is the starting value of the integrator. For most systems involving time, this is zero.

By default, \( \text{init} = 0 \) and \( x0 = 0 \). If the default is to be used, these arguments can be dropped from specification of the differential equation.

For example:

```
MODEL_PREDICTION {
  DEQ{
    RATEIN = if(T >= TLAG) then GUT * KA
            else 0
    GUT : { deriv =(- RATEIN), init = 0, x0 = 0 }
    CENTRAL : { deriv =(RATEIN - CL * CENTRAL / V) }
  }
  CC = CENTRAL / V
} # end MODEL_PREDICTION
```

4.10.2 On Tlag and Bioavailability

Since MDL has no reserved variable names, there is no mechanism for target software to identify lagtime and bioavailability. Models with lag times and bioavailability must use the COMPARTMENT sub-block to specify these input attributes.

4.10.3 COMPARTMENT Sub-block

The COMPARTMENT sub-block is intended to provide the user with a modular approach to describe PK processes through definition of the drug input, distribution, and elimination processes. The functions defined are influenced by the PK macros approach in Monolix. The table below shows how the Compartment definitions in MDL correspond to PK Macros as defined in Monolix.
The major differences over the implementation in Monolix are that in MDL, the “from” and “to” attributes define the links between compartments and processes and that there are no reserved names for compartments or variables e.g. using “K12” and “K21” as variable names confers no special meaning to the use of these variables.

The current version of MDL translates COMPARTMENT specifications into differential equations in NONMEM and to PK Macros in Monolix. Future versions may be able to identify systems of compartments and translate these to closed-form solutions in target software. COMPARTMENT definitions are translated into PK Macros in Monolix and where possible they are mapped to ADVAN closed-form solutions in NONMEM.

COMPARTMENTS sub-block processes are specified as lists with attributes depending on the processes being described.

4.10.3.1 Input & absorption

There are two COMPARTMENT block processes describing input to the system (typically drug input). These are direct and depot. direct defines bolus or zero-order input processes, while depot describes first-order, zero-order or transit chain drug input processes.

Input format is of the form:

`<VARIABLE_NAME> : { type is <depot / direct>,
    to = <VARIABLE>,
    <other arguments> }`

The other arguments depend on the process being described. The table below describes the possible combinations of attributes for different input and absorption processes.

<table>
<thead>
<tr>
<th>Compartment Type</th>
<th>Attribute Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct</td>
<td>to, modelDur(O), modelRate(O), tlag(O), finput(O)</td>
</tr>
<tr>
<td>Depot</td>
<td>to, ka, tlag(O), finput(O)</td>
</tr>
<tr>
<td></td>
<td>to, modelDur, tlag(O), finput(O)</td>
</tr>
<tr>
<td></td>
<td>to, ka, ktr, mtt</td>
</tr>
<tr>
<td></td>
<td>to, modelDur, ktr, mtt</td>
</tr>
</tbody>
</table>

(O) - Optional attribute

`INPUT_KA : { type is depot, to=CENTRAL, ka=KA, tlag=ALAG1, finput=F1}`
4.10.3.2 Distribution processes

MDL defines drug distribution (movement of drug between compartments) through COMPARTMENT block definitions with type `compartment, distribution`.

“`type is compartment`” is used to define PK compartments which have an associated input and elimination process. Peripheral compartments with “`type is distribution`” will not have “`type is input`” or “`type is elimination`” processes associated with them.

```markdown
<VARIABLE NAME> : { `type` is compartment, `modelCmt`=<number> }
```

The `modelCmt` argument is not used.

For example:

```
CENTRAL : { `type` is compartment, `modelCmt`=2}
```

Compartments where the transfer of drug in and out is defined through model variables have “`type is distribution`”.

```markdown
<VARIABLE NAME> : { `type` is distribution, from = <VARIABLE NAME>, kin = <VARIABLE NAME>, kout = <VARIABLE NAME>, `modelCmt`=<number> }
```

For example, to specify a peripheral compartment in a two compartment PK model:

```
PERIPHERAL : { `type` is distribution, `modelCmt`=3, from=CENTRAL, kin=Q/V2, kout=Q/V3}
```

A “`type is effect`” process provides a means to describe the transfer of amounts from a given compartment to an effect compartment e.g. for use with PD models.

```markdown
<VARIABLE NAME> : { `type` is effect, from = <VARIABLE NAME>, keq = <VARIABLE NAME>}
```

<table>
<thead>
<tr>
<th>Compartment Type</th>
<th>Attribute Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>distribution</td>
<td><code>modelCmt</code>, from, kin, kout</td>
</tr>
<tr>
<td>compartment</td>
<td><code>modelCmt</code></td>
</tr>
<tr>
<td>effect</td>
<td><code>modelCmt</code>, from, keq</td>
</tr>
</tbody>
</table>

4.10.3.3 Elimination and transfer processes

Elimination is defined via a list with “`type is elimination`” and specification of the compartment from which drug is eliminated along with variable names for the volume of distribution in the compartment from which drug is eliminated and the micro constant or apparent clearance from the compartment.

If the amount of eliminated drug is not of interest, it is not necessary to name this process. If this is the case, an anonymous list must be used:

```markdown
:: { `type` is elimination, from = <VARIABLE NAME>, `v` = <VARIABLE NAME>, `<k / cl>` = <VARIABLE NAME>, `modelCmt`=<number> }
```

The `modelCmt` argument is not used.

For example in the one compartment model:

```
:: { `type` is elimination, `modelCmt`=2, from=CENTRAL, `v`=V, cl=CL}
```
Note that the \( v \) argument refers to the volume of distribution in the compartment defined by the \( \text{from} \) argument.

A “\text{type is transfer}” process has also been provided which defines the one-way transfer of drug amounts from one compartment to another.

\[
\langle \text{VARIABLE NAME} \rangle : \{ \text{type is transfer}, \text{from} = \langle \text{VARIABLE NAME} \rangle, \text{to} = \langle \text{VARIABLE NAME} \rangle, \text{kt} = \langle \text{VARIABLE NAME} \rangle, \text{modelCmt} = \langle \text{number} \rangle \}
\]

For example:

\[
\langle \text{VARIABLE NAME} \rangle : \{ \text{type is transfer}, \text{modelCmt} = 2, \text{from} = \text{LATENT}, \text{to} = \text{CENTRAL}, \text{kt} = K23 \}
\]

<table>
<thead>
<tr>
<th>Compartment Type</th>
<th>Attribute Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>elimination</td>
<td>from, ( v ), k</td>
</tr>
<tr>
<td></td>
<td>from, ( v ), cl</td>
</tr>
<tr>
<td></td>
<td>from, ( \text{vm} ), ( \text{km} )</td>
</tr>
<tr>
<td>transfer</td>
<td>from, to, ( \text{kt} )</td>
</tr>
</tbody>
</table>

### 4.11 Combining COMPARTMENTS and DEQ blocks

It is possible to use COMPARTMENTS to describe the input processes for differential equations in the \( \text{DEQ} \) block. Using the “\text{type is depot}” or “\text{type is direct}” compartments allows the user to specify lag time and bioavailability (\( \text{tlag} \) and \( \text{finput} \)) which will translate to appropriate terms in target software e.g. \( \text{ALAGn} \) and \( \text{Fn} \) in NM-TRAN. If the COMPARTMENTS specification is not used then model parameters are treated in a very general way and there is no way of mapping these to target tools to implement these input attributes.

If the COMPARTMENT sub-block is not used then delay absorption processes and/or bioavailability needs to be explicitly encoded.

For example in UseCase4 differential equations are used to describe IV and oral administration. The time of dosing DT is passed either from the data or via \( \text{DATA\_DERIVED\_VARIABLES} \) (see section 2.5):

\[
\text{MODEL\_PREDICTION} \{
\begin{align*}
\text{DT} & \\
\text{DEQ} & \{
\text{RATEIN} = \text{if}(\text{T-DT} \geq \text{TLAG}) \text{then GUT} \times \text{KA} \\
\text{else 0} \\
\text{GUT} & : \{ \text{deriv} = (- \text{RATEIN}), \text{init} = 0, x0 = 0 \} \\
\text{CENTRAL} & : \{ \text{deriv} = (\text{RATEIN} \times \text{FORAL} - \text{CL} \times \text{CENTRAL} / \text{V}), \text{init} = 0, x0 = 0 \} \\
\text{CC} & = \text{CENTRAL} / \text{V}
\end{align*}
\}
\text{# end MODEL\_PREDICTION}
\]

In the above example, note that there is a discontinuity in \( \text{RATEIN} \) which is not well handled with differential equations. Note also that the model does not handle multiple doses since then \( \text{DT} \) (time of dose) and hence \( \text{RATEIN} \) would be reset to zero for each new dose.

To alleviate this problem, we can use COMPARTMENTS with DEQ to handle the input processes correctly. The code below shows how to include lag time and bioavailability in a model using differential equations (UseCase4_2):

\[
\text{MODEL\_PREDICTION} \{
\begin{align*}
\text{DT} & \\
\text{DEQ} & \{
\text{RATEIN} = \text{if}(\text{T-DT} \geq \text{TLAG}) \text{then GUT} \times \text{KA} \\
\text{else 0} \\
\text{GUT} & : \{ \text{deriv} = (- \text{RATEIN}), \text{init} = 0, x0 = 0 \} \\
\text{CENTRAL} & : \{ \text{deriv} = (\text{RATEIN} \times \text{FORAL} - \text{CL} \times \text{CENTRAL} / \text{V}), \text{init} = 0, x0 = 0 \} \\
\text{CC} & = \text{CENTRAL} / \text{V}
\end{align*}
\}
\text{# end MODEL\_PREDICTION}
\]

In the above example, note that there is a discontinuity in \( \text{RATEIN} \) which is not well handled with differential equations. Note also that the model does not handle multiple doses since then \( \text{DT} \) (time of dose) and hence \( \text{RATEIN} \) would be reset to zero for each new dose.

To alleviate this problem, we can use COMPARTMENTS with DEQ to handle the input processes correctly. The code below shows how to include lag time and bioavailability in a model using differential equations (UseCase4_2):
MODEL_PREDICTION {
  COMPARTMENT{
    INPUT_KA   : {type is depot, to=CENTRAL, ka=KA, finput=FORAL, tlag=TLAG}
    INPUT_CENTRAL : {type is direct, to = CENTRAL}
  }
  DEQ{
    CENTRAL : { deriv =(- CL * CENTRAL / V), init = 0, x0 = 0 }
  }
  CC = CENTRAL / V
} # end MODEL_PREDICTION

(Nota that this model is not exactly equivalent to UseCase4 which does not allow for superposition of dosing). Note the first-order input to the CENTRAL compartment (INPUT_KA) is “type is depot” while the bolus or zero-order rate input (INPUT_CENTRAL) is “type is direct”. The model corresponding to UseCase4 is shown below (UseCase4_3). In this case there is an explicit differential equation for the depot compartment (GUT)

MODEL_PREDICTION {
  COMPARTMENT{
    INPUT_KA   : {type is direct, to = GUT, finput=FORAL, tlag=TLAG}
    INPUT_CENTRAL : {type is direct, to = CENTRAL}
  }
  DEQ{
    GUT : { deriv =(- GUT * KA), init = 0, x0 = 0 }
    CENTRAL : { deriv =GUT * KA - CL * CENTRAL / V), init = 0, x0 = 0 }
  }
  CC = CENTRAL / V
} # end MODEL_PREDICTION

In the code above, note that both administrations use “type is direct”, but the oral administration (INPUT_KA) has finput and tlag specified. In contrast to the above, the direct input would exactly correspond to the DEQ specification in UseCase4, however the TLAG in UseCase4 is treated as a general parameter, while the TLAG in UseCase4_3 will be translated to ALAGn in NMTRAN.

4.12 OBSERVATION Block

The OBSERVATION block provides the definition of the outcome variable using the prediction from the MODEL_PREDICTION block and/or in the case of continuous data RANDOM VARIABLE_DEFINITION at the observation level. Any calculations or equations needed for this definition should be placed in the MODEL_PREDICTION block can be defined in the OBSERVATION block e.g. calculation of weights for "type is userDefined" error models.

If more than one outcome is specified then we specify each outcome separately within the OBSERVATION block. It is not necessary to have conditional assignment of outcomes to a single outcome variable depending on the variable with “use is divid”. Conditional assignment to outcome definitions is possible if the outcome depends on a covariate or calculated variable. However it is not necessary to conform multiple outcomes to a single observation variable name e.g. Y.

In the current version of MDL, only the standard residual error functions defined below are supported. Equation based definitions of the outcomes are not supported.
4.12.1 Continuous outcomes

For continuous outcomes the `OBSERVATION` block defines how a variable from the `MODEL_PREDICTION` block and `RANDOM_VARIABLE_DEFINITION` providing the residual unexplained random variables are combined in a function to define the outcome.

The mathematical representation of the outcome variable is (after Lavielle, 2014)

$$ h(y_{ij}) = h(f(x_{ij}, \psi_i)) + g(f(x_{ij}, \psi_i), \xi) \epsilon_{ij} $$

Where

$$ y_{ij} = jth \text{ observation for subject } i $$

$$ h = \text{Transformation of the outcome to ensure that the resulting function is an additive function of } f \text{ and } g. \text{ Specified in MDL error models described below using "trans is <h>". Left hand side transformation is specified using the Boolean lhsTrans.} $$

$$ f = \text{structural model prediction from the MODEL_PREDICTION block. Specified in the MDL error models described below through the "prediction" argument.} $$

$$ g = \text{functional definition of the residual error model. Specified in the MDL error models through the "type is additiveError | proportionalError | combinedError1 | combinedError2"} $$

$$ \psi_i = \text{individual parameters defined in the INDIVIDUAL_VARIABLES block} $$

$$ x_{ij} = \text{covariates and regression variables e.g. time, concentration etc.} $$

$$ \xi = \text{parameters of the residual error model defined in the VARIABILITY_PARAMETERS block and referred to in the appropriate "additive" and "proportional" arguments.} $$

$$ \epsilon_{ij} = \text{residual error defined in RANDOM_VARIABLE_DEFINITION block and referred to in the MDL error models described below through the "eps" argument.} $$

The syntax for definition of continuous outcome variables is

<OUTCOME VARIABLE NAME> : {<type is additiveError | proportionalError | combinedError1 | combinedError2 | userDefined,}

<additional arguments defined in table below> }

The following residual error model functions are defined, as described in MDL Language Reference section 2.21 and reiterated here.

<table>
<thead>
<tr>
<th>Name</th>
<th>Return Type</th>
<th>Argument name</th>
<th>Argument Types</th>
</tr>
</thead>
<tbody>
<tr>
<td>additiveError</td>
<td>Real</td>
<td>trans (Optional)</td>
<td>Builtin</td>
</tr>
<tr>
<td></td>
<td></td>
<td>lhsTrans</td>
<td>Boolean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>additive</td>
<td>Real</td>
</tr>
<tr>
<td></td>
<td></td>
<td>prediction</td>
<td>Real</td>
</tr>
<tr>
<td>proportionalError</td>
<td>Real</td>
<td>trans (Optional)</td>
<td>Builtin</td>
</tr>
<tr>
<td></td>
<td></td>
<td>lhsTrans</td>
<td>Boolean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>proportional</td>
<td>Real</td>
</tr>
<tr>
<td></td>
<td></td>
<td>prediction</td>
<td>Real</td>
</tr>
<tr>
<td></td>
<td></td>
<td>eps</td>
<td>Real</td>
</tr>
</tbody>
</table>
combinedError1 defines the following model:

\[ h(y_{ij}) = h(f(x_{ij}, \psi_i)) + (a + bf(x_{ij}, \psi_i))e_{ij} \]

combinedError2 defines the following model:

\[ h(y_{ij}) = h(f(x_{ij}, \psi_i)) + \sqrt{a^2 + b^2f(x_{ij}, \psi_i)}e_{ij} \]

the lhsTrans Boolean argument allows the user to apply transformations to the DV without having to transform the data column prior to analysis.

As with the \{type is linear, \} and \{type is general, \} definitions in the INDIVIDUAL_VARIABLES block, we use defined types here to make explicit the relationships between predictions and residual random variable terms to facilitate interoperability between target software. A more general equation form could be used, but this would not necessarily translate successfully to all target software for estimation.

Use \{type is userDefined, \} to specify an arbitrary relationship between prediction, the residual error random variable which is typically Normal(0,1), and the with associated function g(\cdot) defined above. Using this form ensures that correct calculation of the weighted residuals can be calculated.

The current version of MDL does not support definition of outcomes with arbitrary functions of variables and random variables. Any equations written in the OBSERVATION block are treated as variables to be used in definition of the observation through a list definition as described above (including UserDefined). If a list definition is not used, then the observation equation may not be translated correctly to the target software tool.

4.12.2 Discrete data

Discrete data outcomes are described by referencing a suitable distribution for the outcome. In this version of MDL we assume that the parameters of the relevant distributions are supplied either in the data, for example the number of trials, N, in a binomial distribution, or are defined in the MODEL_PREDICTION block.
In this version of MDL we assume an identity link for all models - that is the parameter supplied to the distribution must be on the appropriate scale for that distribution - the Poisson rate parameter must have a positive value, probabilities for binary and categorical distributions must be on the scale (0,1).

Count, discrete, categorical outcomes must be specified within the RANDOM_VARIABLE_DEFINITION block for the dv level via a suitable ProbOnto distribution (see also section 0). The random variable is then declared in the OBSERVATION block via an anonymous list, as has been seen previously when defining individual variables via the RANDOM_VARIABLE_DEFINITION block. Since continuous outcomes have a residual error specified at the DV level, it is inferred that the outcome defined in the OBSERVATION block is at the DV level of variability. However for other types of data, it is less clear that this is the case. Thus we must use RANDOM_VARIABLE_DEFINITION to define these outcomes using ProbOnto definitions and then provide additional information in the OBSERVATION block to assign additional attributes.

The syntax is as follows:

```
RANDOM_VARIABLE_DEFINITION(level=DV){
  <outcome variable> ~ <ProbOnto distribution>
}

OBSERVATION{
  :: {type is <count/discrete/categorical>, variable = <outcome variable>}
}
```

See below for examples pertaining to specific outcome types.

### 4.12.2.1 Count data

For count data, we have the following syntax:

```
RANDOM_VARIABLE_DEFINITION(level=DV){
  <variable> ~ <ProbOnto distribution for count data e.g. Poisson1>
}

OBSERVATION{
  :: {type is count, variable = <variable>}
}
```

For example (also showing the appropriate INDIVIDUAL_VARIABLES, MODEL_PREDICTION and OBSERVATION blocks) (UseCase11)

```
INDIVIDUAL_VARIABLES{
  BASECOUNT : {type is linear, trans is ln, pop = POP_BASECOUNT, ranEff = eta_PPV_EVENT }
  BETA = POP_BETA
}

MODEL_PREDICTION{
  lnLAMBDA=ln(BASECOUNT) + BETA*CP
  LAMBDA = exp(lnLAMBDA)
}

RANDOM_VARIABLE_DEFINITION(level=DV){
  Y ~ Poisson1(rate=LAMBDA)
}
```
Note in the above example that the BASECOUNT variable is specified using the linear function and a natural log transformation on both sides to ensure that BASECOUNT is positive. The linear relationship with CP (plasma concentration) is defined within the MODEL_PREDICTION block. We cannot use CP as a covariate in the linear(...) function as CP varies with time and so is regarded as a regressor rather than a covariate. In UseCase11 since there is no model for the pharmacokinetics we use CP as the independent variable (IDV) in the model and “use is idv” in the DATA_INPUT_VARIABLES block. We also take exponential of lnLAMBDA to ensure that the variable LAMBDA is on the positive scale before using this in the Poisson distribution.

This is an example where a little consideration of the random effects and model prediction can facilitate interoperability. Writing an equation for the INDIVIDUAL_VARIABLES we may have defined

INDIVIDUAL_VARIABLES{
  \ln LAMBDA = \ln(POP\_BASECOUNT) + BETA*CP + \eta_{PPV\_EVENT}
}

Using this formulation of the model though would not guarantee interoperability with some target software for estimation since the equation for lnLAMBDA is user-defined.

In the current version of MDL, only the Poisson distribution can be used to specify count data. In future versions of MDL many more distributions will be available through the Prob-Onto distribution specification in PharmML.

There are many distributions defined in ProbOnto which will describe count data. The diagram below illustrates a few of these and relationships between them as described in the ProbOnto Knowledge Base (www.probonto.org)
4.12.2.2 Binary data

Similar to count data above, we define the binary outcome and its distribution in a RANDOM_VARIABLE_DEFINITION block at the observation level of variability. Note how we define the names of the categories on the left hand side, and then the probability distribution defines the likelihood of the second category.

```
RANDOM_VARIABLE_DEFINITION(level=DV)
    Y withCategories {<category1>,<category2>}
        ~ <ProbOnto distribution e.g. Bernoulli | Binomial>
}
```

For example (again, showing the INDIVIDUAL_VARIABLES, MODEL_PREDICTION and OBSERVATION blocks to show the model construction):

```
RANDOM_VARIABLE_DEFINITION(level=ID)
    eta_PPV_EVENT ~ Normal(mean=0, var=PPV_EVENT )
}# end RANDOM_VARIABLE_DEFINITION

INDIVIDUAL_VARIABLES{
    indiv_BASE : {type is linear, pop= POP_BASEP,
                    ranEff=[eta_PPV_EVENT], trans is logit}
}# end INDIVIDUAL_VARIABLES

MODEL_PREDICTION{
    LP = logit(indiv_BASE) + POP_BETA*CP
    P1 = invLogit(LP)
}# end MODEL_PREDICTION

RANDOM_VARIABLE_DEFINITION(level=DV)
    Y withCategories {none, event}   ~ Bernoulli1(probability=P1)
}

OBSERVATION{
    :: {type is discrete, variable = Y }
}# end ESTIMATION
```

Note that the INDIVIDUAL_VARIABLES block defines the individual baseline by combining the population parameter and the random effect. Note also that this specification uses a logit transformation to ensure that the individual baseline indiv_BASE variable is on the (0,1) probability scale. Then, the linear regression with plasma concentration (CP) is defined in the MODEL_PREDICTION - Note that CP is not a covariate. Finally LP is back-transformed to the probability scale to give variable P1 which is the probability of an event to be used in the Bernoulli distribution. By defining how the 0,1 in the data correspond to named events {none, event} it is easier to understand exactly what category is being modelled.

An alternative distribution for the same model is the Binomial distribution with one trial:

```
RANDOM_VARIABLE_DEFINITION(level=DV)
    Y withCategories {none, event}   ~ Binomial1(numberOfTrials=1, probability=P1)
}

OBSERVATION{
    :: {type is discrete, variable = Y }
} # end of OBSERVATION
4.12.2.3  Categorical data

Again, similar to count and binary data, the syntax for Categorical data outcomes is:

\[
\text{RANDOM_VARIABLE_DEFINITION(level=DV)}\{
  \langle\text{variable}\rangle \text{ withCategories}\{ \langle\text{category1}\rangle, \ldots, \langle\text{categoryk}\rangle \} \}
\sim \langle\text{ProbOnto distribution for categorical data e.g. CategoricalNonordered1 | CategoricalOrdered1}\rangle
\}
\]

\[
\text{OBSERVATION}\{
  :: \{\text{type is categorical, variable=\langle\text{variable}\rangle}\}
\}
\]

For example (UseCase13_1):

\[
\text{GROUP_VARIABLES}\{
  B0 = \text{Lgt0}
  B1 = B0 + \text{Lgt1}
  B2 = B1 + \text{Lgt2}
\}
\]

\[
\text{INDIVIDUAL_VARIABLES}\{
  \text{indiv}_B0 : \{\text{type is general, grp=B0, ranEff = eta_PPV_EVENT}\}
  \text{indiv}_B1 : \{\text{type is general, grp=B1, ranEff = eta_PPV_EVENT}\}
  \text{indiv}_B2 : \{\text{type is general, grp=B2, ranEff = eta_PPV_EVENT}\}
\}
\]

\[
\text{MODEL_PREDICTION}\{
  \text{EDRUG} = \text{Beta} \ast \text{CP}
  A0 = \text{indiv}_B0 + \text{EDRUG}
  A1 = \text{indiv}_B1 + \text{EDRUG}
  A2 = \text{indiv}_B2 + \text{EDRUG}
  \text{P0} = \text{invLogit}(A0)
  \text{P1} = \text{invLogit}(A1)
  \text{P2} = \text{invLogit}(A2)
  \text{Prob0} = \text{P0}
  \text{Prob1} = \text{P1} - \text{P0}
  \text{Prob2} = \text{P2} - \text{P1}
  \text{Prob3} = 1 - \text{P2}
\}
\]

\[
\text{RANDOM_VARIABLE_DEFINITION(level=DV)}\{
  Y \text{ withCategories}\{ \text{none, mild, moderate, severe } \} \sim \text{CategoricalOrdered1(categoryProb=[Prob0, Prob1, Prob2, Prob3])}
\}
\]

\[
\text{OBSERVATION}\{
  :: \{\text{type is categorical, variable=Y}\}
\}
\]
In the above code, the cutpoints between categories are defined in the \texttt{GROUP_VARIABLES} block (B0, B1, B2) and individual values for these are defined in the \texttt{INDIVIDUAL_VARIABLES} block. The linear effect of CP (plasma concentration) is defined in the \texttt{MODEL_PREDICTION} block and this is added to the individualised cutpoints (A0, A1, A2). These are then back-transformed to the probability scale (P0, P1, P2) and the ordered categorical model is defined by calculating the probability of each category as the difference from the previous category - Prob0, Prob1, Prob2, Prob3.

### 4.12.3 Time to event data

Time to event (TTE) models are modelled by specifying the hazard function. The PharmML to target software tool converters handle the translation of the hazard specification to target tool implementation. For some software this involves calculation of the survival function and associated likelihood.

For an arbitrary hazard function \( \lambda(t) \):

<table>
<thead>
<tr>
<th>Hazard function</th>
<th>( \lambda(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative hazard function</td>
<td>( \Lambda(a,b) = \int_a^b \lambda(t)dt )</td>
</tr>
<tr>
<td>Survival function</td>
<td>( P(T &gt; t) = e^{-\Lambda(t_0,t)} )</td>
</tr>
<tr>
<td>Probability density function</td>
<td>( p(t) = \lambda(t)e^{-\Lambda(t_0,t)} )</td>
</tr>
<tr>
<td>Cumulative distribution function</td>
<td>( P(T &lt; t) = \int_0^t p(s)ds )</td>
</tr>
</tbody>
</table>

For an introduction to TTE models see Holford (2013) and for a tutorial in implementation in NONMEM and Monolix see Holford and Lavielle, (2011).

The MDL syntax for time to event outcomes is:

```
<OUTCOME VARIABLE NAME> : {type is tte,
                         hazard = <VARIABLE>
                         }
```

For example (UseCase14.mdl):

```
INDIVIDUAL_VARIABLES{
    BTATRT = POP_BTATRT
    H_BASE = POP_HBASE
}

MODEL_PREDICTION{
    HBASE=H_BASE/365
    HAZTRT=BTATRT*TRT
    HAZ = HBASE * (1+HAZTRT)
}# end MODEL_PREDICTION

OBSERVATION{
    Y : {type is tte, hazard = HAZ }
```
In the above case the hazard and the effect of treatment on the hazard is calculated in the 
**GROUP_VARIABLES** block. This is then used in the **MODEL_PREDICTION** block to calculate the hazard for the event. The model outcome variable Y is then defined as having type **tte** and the hazard calculated in the **MODEL_PREDICTION** block is passed in as an argument. Specification of the model is then very simple for the user - no calculation of Survival functions nor likelihood is necessary.

In the current MDL, TTE models are able to be handled equally by NONMEM and Monolix. To facilitate this, we impose certain constraints on dataset conventions. There must be a data record at the start of the interval during which the hazard will be integrated. We use DV = 0 to denote right censoring and DV = 1 to denote an event.

**Currently only exact time of event and right censoring is supported in MDL. Future versions will support interval censoring and repeated time to event.**

The convention in NONMEM datasets of using MDV to identify the start of the observation period for assessing TTE cannot be used. In order to make the model and data interoperable the user must ensure that MDV is not included in the data or use “**use is ignore**” for MDV in the **DATA_INPUT_VARIABLES** block.

### 4.13 FUNCTIONS block

This block allows users to define their own functions, for example for use with interpolation.

The syntax is as follows:

```text
<function name> :: function(<argument1> :: <argument1 type>,
    <argument2> :: <argument2 type>,
    ...
    <argumentk> :: <argumentk type>)
:: <function result type>
is
<expression using argument1... argumentk>
```

Specifying the types of each argument and the type of the function result allows validation of the function inputs and outputs.

To call the function, the user types “&” before the function name.

For example:

```text
FUNCTIONS{
    myInterp::function(t::real, x0::real, t0::real, x1::real, t1::real)::real is x0
}
```

```text
DATA_INPUT_VARIABLES {
    ID : { use is id }
    TIME : { use is idv }
    WT : { use is covariate, interp=&constInterp }
    AGE : { use is covariate, interp=&myInterp }
    ...
}
```
4.14 **POPULATION_PARAMETERS** block

The **POPULATION_PARAMETERS** block allows the user to define models and model parameters that exist at the population level. This may be used in hierarchical models to define population parameters when there are additional levels of hierarchy above the individual.

In most “population approach” models, the individuals are assumed to be drawn from a population, the characteristics of which are described through fixed and random effect models. Inferences are then made on the “population parameters” which we take to be representative of the population from which the individuals are drawn.

In meta-analysis across many studies, regions, demographic populations it may be useful to characterise additional levels of hierarchy characterising how individuals within each study, region or demographic differ systematically (through fixed effect models) and randomly (through variability models). These higher level models can be expressed in the **POPULATION_PARAMETERS** block.

In the current MDL, population models are defined through combination of **RANDOM_VARIABLE_DEFINITION** and **POPULATION_PARAMETERS** definitions. In the current MDL expressions (equations) are NOT allowed in the **POPULATION_PARAMETERS** block.

The **POPULATION_PARAMETERS** block uses random variables defined in the **RANDOM_VARIABLE_DEFINITION** block. The syntax for the **POPULATION_PARAMETERS** block is as follows:

```
POPULATION_PARAMETERS{
  :: { type is <continuous|categorical>,
      variable=<RANDOM_VARIABLE_DEFINITION variable> }
}
```

For example (/FourModels/Hierarchical_Model.mdl):

```
RANDOM_VARIABLE_DEFINITION(level=POP){
  w_pop ~ Normal(mean = ws, sd = gw)
  V_pop ~ Normal(mean = Vs, sd = gV)
} # end RANDOM_VARIABLE_DEFINITION

POPULATION_PARAMETERS{
  :: { type is continuous, variable=w_pop
  :: { type is continuous, variable=V_pop }
}

RANDOM_VARIABLE_DEFINITION(level=ID) {
  ETA_BSV_V ~ Normal(mean = 0, sd = omega_V)
} # end RANDOM_VARIABLE_DEFINITION

INDIVIDUAL_VARIABLES {
  V : { type is linear, trans is ln, pop=V_pop,
       fixEff={coeff=BETA_WT, cov=WT}, ranEff=ETA_BSV_V
  } # end INDIVIDUAL_VARIABLES

MODEL_PREDICTION {
  D
  f = D/V * exp(-k*T)
} # end MODEL_PREDICTION
```

In the model above, the mean weight for each population is drawn from a Normal random variable. The mean Volume of distribution also varies for each population and is similarly drawn from a Normal
random variable. The parameters for inference would be $V_s$ (“global” mean of Volume of distribution),
gV (between population variability), $\omega_V$ (between individual variability), BETA_WT (fixed effect
of Weight). $V_{\text{pop}}$ gives the population prediction of Volume of distribution for each population
observed in the data, while $V$ gives the individual predicted Volume of distribution.
5 The Task Properties Object

The Task Properties Object is intended to convey information to specific target software related to algorithms, settings, options for performing a given task. In the current version of MDL, the implementation of this is very limited. Some Task Properties Object settings are generic (apply across different target software) but most are specific to the intended target software.

Task properties are specific to the task e.g. ESTIMATE, SIMULATE, OPTIMISE, EVALUATE. TARGET_SETTINGS are specific to a given target software tool. If the user intends to estimate with a given tool but no ESTIMATE block is given in the Task Properties Object and/or if the task properties are not specified for the intended tool, then default settings are used.

In the current version of MDL, only the estimation algorithm property can be set.

5.1 Intended use of Task Properties

The user should provide information about settings and options for each target software that they wish to use for a given (estimation) task. These settings and options might be specified to ensure reproducibility of results regardless of target software i.e. to ensure that the results from a given target software are comparable with results from different target software.

Alternatively, the user may wish to provide settings and options that they will frequently use with a given target software - so that every time they estimate with a given target they use the same settings.

The modularity of MDL allows the user to preset or reuse Task Properties objects between models. The user may then have preferred Task Properties for estimation that can be called upon during the relevant points in their M & S workflow.

As with other MDL blocks, it is possible to specify multiple Task Properties blocks within a single .mdl file and then reference only the one required for use with a specific task in either the MOG Object or via R. This allows the user to specify preferred settings for multiple target software tools - facilitating reproducibility and reuse. Even though many Task Properties Objects can be given in an .mdl file, the MOG Object should contain only ONE Task Properties Object.

5.2 Why use Task Properties for settings and options rather than arguments of functions in the ddmore R package?

Task Properties are distinct from arguments to the ddmore R package functions for executing tasks - the former passes information to the appropriate target software about the particular settings and options required for a given task. The ddmore function arguments are equivalent to the command line settings or options which are employed when invoking target software.

For example, we may use Task Properties to define the estimation algorithm and associated settings with NONMEM, but define command line options for PsN which govern how NONMEM should be called by PsN.

5.3 How are Task Properties used by MDL and PharmML?

The Task Properties generic items are parsed and understood by the MDL editor within the MDL-IDE. However target software specific settings and options are not parsed by the MDL-IDE - these are passed as is via the PharmML to the target software converter where they are interpreted and converted where they may appear in target software code, external settings or options files as appropriate.

5.4 ESTIMATE Block

In the current MDL version, the only block supported is the ESTIMATE block. The syntax for this block is:
As stated previously, this block will be extended in future versions of MDL to capture target software specific settings and options.

5.4.1 TARGET_SETTINGS

The TARGET_SETTINGS block holds specific settings for the given target software. These settings may be given in name-value pairs within the TARGET_SETTINGS block, or they may be specified in an external file.

MDL does not check whether the specified options within the TARGET_SETTINGS block are appropriate for a given tool. This is performed by the converter for the target tool. See the MDL Reference section for a list of which options are available for each target software.

The syntax for the TARGET_SETTINGS block is:

```
TARGET_SETTINGS(target="NONMEM" | "MONOLIX" | "BUGS" | "PFIM", settingsFile = "<filename>"){ 
    set <target specific setting name> = <target specific setting value>,
    <target specific setting name> = <target specific setting value>,
    ...
}
```

Note the comma separation between elements of the set statement.

For example (UseCase1_1):

```
SAEM_task = taskObj {
    ESTIMATE{
        set algo is saem
        TARGET_SETTINGS(target="NONMEM"){
            # $EST METHOD=SAEM INTERACTION NBURN=3000 NITER=500 PRINT=100 SEED=1556678 ISAMPLE=2
            # $COV MATRIX=R PRINT=E UNCONDITIONAL SIGL=12
            set INTER=true, NBURN=3000, NITER=500, PRINT=100, SEED=1556678, ISAMPLE=2,
            COV_=true, COV_MATRIX="R", COV_PRINT="E", COV_UNCONDITIONAL=true,
            COV_SIGL=12
        }
    }
}
```

```
TARGET_SETTINGS(target="MONOLIX", settingsFile=["tables.xmlx"]){
    set graphicsSettings="tables.xmlx"
}
```

```
BUGS_task = taskObj{
    ESTIMATE{
        set algo is mcmc
        TARGET_SETTINGS(target="BUGS"){
            set nchains = 1, # Number of MCMC chains
            burnin = 1000, # Number of MCMC Burn-in iterations
            niter = 5000, # Number of iterations
            parameters = "V,CL,KA,TLAG", # Parameters to monitor
            odesolver="LSODA", # or "RK45"
            winbugsgui ="false" # or "true"
        }
    }
}
```
FOCEI_task= taskObj {
    ESTIMATE{
        set algo is focei
        TARGET_SETTINGS(target="NONMEM"){
            # $EST METHOD=COND INTERACTION MAXEVAL=9999 NSIG=3 SIGL=10 PRINT=5 NOABORT NOPRIOR=1
            # FILE=example1.ext
            # $COV MATRIX=R PRINT=E UNCONDITIONAL SIGL=12
            set INTER=true, MAXEVAL=9999, NSIG=3, SIGL=10, PRINT=5, NOABORT=true,
            NOPRIOR=1, FILE="example1.ext",
            COV_MATRIX="R", COV_PRINT="E", COV_UNCONDITIONAL=true, COV_SIGL=12,
            MSFO_FILE="MSFO.msf"
        }
    }
} # end of task object

For BUGS, the following properties are valid (as shown above):

- nchains = <integer>
- niter = <integer>
- parameters = <comma-separated character string of parameter names>
- odesolver = <"LSODA" | "RK45”>
- winbugsgui = <"true" | "false”>

For NONMEM, the following rules are used in the TARGET_SETTINGS block:

a) Individual properties influence only $EST statement or the $COV

b) Properties starting with the string ‘COV_’ will be added to the $COV statement using the following rules

   a. The string ‘COV_’ will be removed from the property

   b. If the remainder of the property is a string/integer assignment, it will be added as a parameter on the $COV statement verbatim (with any unnecessary quotes removed)

   c. If the remainder of the property is a boolean assignment resolving to true, just the parameter name will be added on the $COV statement verbatim

c) If there are any ‘COV_’ properties present, then a $COV statement will be generated. Otherwise it will not be generated

d) Any property starting with a string that is not ‘COV_’ will be added to the $EST statement using the same string/integer/boolean rules as for $COV
6 The Prior Object

For Bayesian estimation, the user must specify a Prior Object, which is used in place of the Parameter Object. The Prior Object defines prior distributions or values for all model parameters - both structural and variability. The prior distributions form an additional level of model hierarchy above all other levels of variability in the model. This level of the hierarchy does not need to be explicitly defined in the Model Object VARIABILITY_LEVELS block. It is implied through use of the Prior Object.

6.1 Prior distributions vs initial values vs fixed values

At present we do not support translation of the Prior Object to NONMEM prior specification for use with the $BAYES estimation algorithm. The only supported Bayesian estimation tool currently is WinBUGS. While NONMEM accepts a mix of prior distribution specification and initial values for estimation, WinBUGS requires prior distributions for all parameters, or fixing parameters to a given value. A fixed value can be thought of as a probability mass function (pmf) on a single value. A fixed value for a parameter represents a very strong prior on the value of that parameter. Bounds on parameters should be handled using an appropriate ProbOnto distribution e.g. Beta, Gamma, Uniform, Half-Normal, Truncated-Normal.

6.2 PRIOR_PARAMETERS Block

The PRIOR_PARAMETERS block holds constants which may be used in the PRIOR_VARIABLE_DEFINITION or NON_CANONICAL_DISTRIBUTION blocks. This allows the user to specify the general form of the prior distribution in the PRIOR_VARIABLE_DEFINITION block and then examine sensitivity to prior choice by altering the values in the PRIOR_PARAMETERS.

The PRIOR_PARAMETERS block should contain variable assignment statements.

<VARIABLE> = <value>

Note that in the PRIOR_PARAMETERS block, the variable is assigned a value, not a list with attributes. As mentioned above, if a model parameter is to be fixed then it takes the value assigned in the PRIOR_PARAMETERS block. An attribute “fix=true” is not required.

For example (/Priors/UseCase1_PRIOR):

PRIOR_PARAMETERS{
  # prior on "THETA"
  MU_POP_CL = 0.2
  MU_POP_V  = 10
  MU_POP_KA = 0.3
  MU_POP_TLAG = 0.75
  VAR_POP_CL = 1
  VAR_POP_V  = 1
  VAR_POP_KA = 1
  VAR_POP_TLAG = 0.1

  # prior on "OMEGA"
  MU_R_CL  = 0.2
  MU_R_V   = 0.2
  MU_R_V_CL = 0
  DF_OMEGA = 2

  MU_OMEGA_KA = 1
  MU_OMEGA_TLAG = 1

  # prior on "SIGMA"
}
a_POP_RUV_ADD = 1.1
b_POP_RUV_ADD = 3
a_POP_RUV_PROP = 1.1
b_POP_RUV_PROP = 3

} # end PRIOR_PARAMETERS

6.3 PRIOR_VARIABLE DEFINITION - Parametric distributions as priors

In the PRIOR_VARIABLE DEFINITION block we set up the prior distributions for the STRUCTURAL and VARIABILITY parameters of the Model Object. All model parameters must have a prior distribution specified, or a constant value set.

If model parameters are correlated or have a multivariate distribution then it is common (although not mandatory) to specify multivariate prior distributions. To do so, the user is likely to need to specify vectors of means and matrices for covariances or correlations. The syntax for specifying vectors and matrices is given in section 9.1.4.7.

The PRIOR_VARIABLE DEFINITION block can contain assignment, transformation and random variable definitions using ProbOnto definitions for distributions.

For example (UseCase1_PRIOR):

PRIOR_VARIABLE DEFINITION{
    # prior on "THETA"
    lMU_POP_CL = ln(MU_POP_CL)
    lPOP_CL ~ Normal(mean=lMU_POP_CL, var=VAR_POP_CL)
    POP_CL = exp(lPOP_CL)

    lMU_POP_V = ln(MU_POP_V)
    lPOP_V ~ Normal(mean=lMU_POP_V, var=VAR_POP_V)
    POP_V = exp(lPOP_V)

    lMU_POP_KA = ln(MU_POP_KA)
    lPOP_KA ~ Normal(mean=lMU_POP_KA, var=VAR_POP_KA)
    POP_KA = exp(lPOP_KA)

    lMU_POP_TLAG = ln(MU_POP_TLAG)
    lPOP_TLAG ~ Normal(mean=lMU_POP_TLAG, var=VAR_POP_TLAG)
    POP_TLAG = exp(lPOP_TLAG)

    # priors on "OMEGA"
    R_mat =       
                         [[ MU_R_CL, MU_R_V_CL; 
                            MU_R_V_CL, MU_R_V ]]
    TAU_CL_V ~ Wishart2(inverseScaleMatrix=R_mat, degreesOfFreedom=DF_OMEGA)
    OMEGA_CL_V = inverse(TAU_CL_V)
    PPV_CL = sqrt(OMEGA_CL_V[1,1])
    PPV_V = sqrt(OMEGA_CL_V[2,2])
    PPV_V_CL = OMEGA_CL_V[1,2]
    TAU_KA ~ Gamma2(shape=0.001, rate=0.001)
    PPV_KA = sqrt(1/TAU_KA)

    # prior on "SIGMA"
    invRUV_ADD ~ Gamma2(shape=a_POP_RUV_ADD, rate=b_POP_RUV_ADD)
    invRUV_PROP ~ Gamma2(shape=a_POP_RUV_PROP, rate=b_POP_RUV_PROP)
RUV_ADD = sqrt(1/invRUV_ADD)
RUV_PROP = sqrt(1/invRUV_PROP)
}

Note the following parameters have prior distributions assigned: POP_CL, POP_V, POP_KA, POP_TLAG, PPV_CL, PPV_V, COV_CL_V, PPV_KA, RUV_ADD, RUV_PROP.

Note that priors on between subject variability are given on the precision scale (= 1 / variance). This is to facilitate use in BUGS. Here the distributions used are Wishart and Gamma on the precision parameters, but Inverse-Wishart and Inverse-Gamma may alternatively be used on the variance-covariance matrix and variance parameters.

Note also that priors for POP_CL, POP_V, POP_KA and POP_TLAG could also be defined using logNormal1 distributions (using the ProbOnto distribution) instead of transforming and back-transforming using Normal(…) distributions.

To specify a matrix, we use double square brackets, and specify elements row-wise, separated with a semi-colon. To specify elements of a matrix we use the R convention of square bracket specifying row and column entries. For example:

\[
R_{\text{mat}} = \begin{bmatrix}
\text{MU}_R_{\text{CL}}, \text{MU}_R_{\text{V CL}}; \\
\text{MU}_R_{\text{V CL}}, \text{MU}_R_{\text{V}}
\end{bmatrix}
\]

To specify the first row and column entry (corresponding to MU_R_CL): R_{mat}[1,1].

Note that a Gamma distribution is used to define the prior on the between subject variability for PPV_KA. This is a legacy from the early days of fitting hierarchical models in BUGS where Gamma priors were conjugate and easier to sample from. They have been somewhat discredited as prior choices. Recent literature has favoured Half-Cauchy priors on variance parameters of hierarchical models as they are robust against smaller numbers of subjects.\footnote{Gelman, A. Prior distributions for variance parameters in hierarchical models. \textit{Bayesian Analysis} (2006) 1, Number 3, pp. 515-533}

6.4 Non-parametric and empirical distributions as priors - inline data.

As an alternative to parametric distributions as priors, the user can specify non-parametric distributions (specifying bins of values and probabilities for each bin) or empirical distributions (specifying data forming the basis of empirical sampling). Univariate and multivariate sampling distributions have been defined in MDL for non-parametric and empirical sampling distributions.

In both cases the source for the non-parametric or empirical sampling can be specified inline via the PRIOR_PARAMETERS block, or by referencing an external data source in the PRIOR_SOURCE block.

6.4.1 Non-parametric distribution specification with inline data.

To specify a non-parametric distribution, MDL has a distributions called NonParametric and MultiNonParametric. These map to the ProbOnto RandomSample non-parametric distribution definition. To specify the non-parametric distributions, the user must supply bins and probabilities for sampling. To specify this inline we create a vector (for NonParametric) or matrix (for MultiNonParametric) of bins and a vector of probabilities. These are specified in the PRIOR_PARAMETERS block.

For example (Priors examples, Example3421dep)

\begin{verbatim}
PRIOR_PARAMETERS{
\end{verbatim}
...  
# For Non-Parametric distribution
bins_Pop_K_V =
matrix(vector=[2.006510, 2.045465, 2.084421, 2.123377, 2.162333, 2.201288, 2.240244, 2.279200, 2.318156, 2.357111, 5.050013, 5.050013, 5.050013, 5.050013, 5.050013, 5.064166, 5.064166, 5.064166, 5.064166, 5.064166, 5.064166, 5.078318, 5.078318, 5.078318], ncol=2, byRow is FALSE)

p_Pop_K_V =
[0.033333, 0.100000, 0.100000, 0.200000, 0.100000, 0.066667, 0.166667, 0.100000, 0.066667, 0.066667]
} # end PRIOR

PRIOR_VARIABLE_DEFINITION{
 # prior on "THETA"
 POP_K_V ~ MultiNonParametric(probability=p_Pop_K_V, bins=bins_Pop_K_V) 
 POP_K = POP_K_V[1]
 POP_V = POP_K_V[2]
 ... 
} # end PRIOR_VARIABLE_DEFINITION

Here a matrix of bins for POP_K and POP_V is created by specifying a vector of values and then defining the number of columns and method for filling the matrix. A vector of probabilities is also defined. Then in the PRIOR_VARIABLE_DEFINITION block the multivariate non-parametric sampling distribution is defined referencing the probabilities and bins. Finally the Priors for POP_K and POP_V are defined by referencing the elements of the POP_K_V vector.

6.4.2 Empirical distribution specification with inline data.

Similarly, the user can specify the data source for empirical sampling within the PRIOR_PARAMETERS block and then refer to this in defining the sampling distribution for the PRIOR_VARIABLE_DEFINITION. For example (Priors examples, Example 3422)

PRIOR_PARAMETERS{
 data_Pop_K_V =
 matrix(vector=[2.006510, 2.045465, 2.084421, 2.123377, 2.162333, 2.201288, 2.240244, 2.279200, 2.318156, 2.357111, 5.050013, 5.050013, 5.050013, 5.050013, 5.050013, 5.064166, 5.064166, 5.064166, 5.064166, 5.064166, 5.064166, 5.078318, 5.078318, 5.078318], ncol=2, byRow is FALSE)
 ... 
} # end PRIOR

#

PRIOR_VARIABLE_DEFINITION{
 # prior on "THETA"
 POP_K_V ~ MultiEmpirical(data=data_Pop_K_V)
 POP_K = POP_K_V[1]
 POP_V = POP_K_V[2]
 ... 
} # end PRIOR_VARIABLE_DEFINITION

Again, the data source is defined by specifying a matrix of values for POP_K and POP_V and then using this as the basis for the MultiEmpirical sampling distribution. For the univariate Empirical sampling distribution, only a vector would be needed as the basis for sampling.
6.5 NON_CANONICAL_DISTRIBUTION block

As an alternative to inline data specification for non-parametric or empirical sampling distributions, the user may reference an external dataset for bins and probabilities (for use with non-parametric sampling distributions) or data for the basis of the empirical sampling distribution.

6.5.1 PRIOR_SOURCE block

Similarly to the SOURCE block within the Data Object, the PRIOR_SOURCE block is a named list providing the file name, format of the source data. However, the PRIOR_SOURCE block adds an argument to the list to provide a vector of column names to be used in the data source for the sampling distributions.

The syntax is as follows:

```
PRIOR_SOURCE{
  <data source name> : { file = <"filename.csv">, inputFormat is csv, column = [<"variable name1", “variable name2”, ... , “variable name k”]}
}
```

Multiple data sources may be defined within the PRIOR_SOURCE block.

6.5.2 INPUT_PRIOR_DATA block

The PRIOR_SOURCE data objects can then be referenced in the INPUT_PRIOR_DATA block to define how the data file columns map to objects to be used in the PRIOR_VARIABLE_DEFINITION block sampling distributions. This is done using anonymous lists.

The syntax is as follows:

```
INPUT_PRIOR_DATA{
  :: { src = <PRIOR_SOURCE data variable>, vector | matrix = <PRIOR_VARIABLE_DEFINITION object>,
       column = "<PRIOR_SOURCE data column name>" }
}
```

For example:

```
NON_CANONICAL_DISTRIBUTION{
  PRIOR_SOURCE{
    NonPar_K_V : { file="Nonparametric_K_V.csv", inputFormat is csv,
                  column= ["bins_k", "bins_v", "p_k_v"]}
    Emp_SIGMA : { file="Empirical_Sigma.csv", inputFormat is csv,
                  column= ["data_SIGMA2"]}
  }

  INPUT_PRIOR_DATA{
    :: { src= NonPar_K_V, vectorVar=p_k_v, column="p_k_v" }
    :: { src= NonPar_K_V, matrixVar=bins_k_v, column= ["bin_k", "bins_v"] }
    :: { src= Emp_SIGMA, column="data_SIGMA2", vectorVar=data_SIGMA2 }
  }

  PRIOR_VARIABLE_DEFINITION{
    p_k_v::vector
    bins_k_v::matrix
    data_SIGMA2::vector
    POP_k_v ~ MultiNonParametric(bins=bins_k_v, probability=p_k_v)
  }
}
```
\[ \text{POP\_SIGMA2} \sim \text{Empirical}(\text{data}=\text{data\_SIGMA2}) \]

\[ \text{POP\_K} = \text{POP\_k\_v}[0] \]
\[ \text{POP\_V} = \text{POP\_k\_v}[1] \]

In the above example, two sources are specified - one giving non-parametric sampling bins and probabilities for K and V, the other providing values for SIGMA to be used in the empirical sampling distribution. In the definition of NonPar_K_V we want to read three columns from the PRIOR\_SOURCE data file - “bins\_k” and “bins\_v” to specify the bins for K and V, and “p\_k\_v” to specify the probabilities for sampling these bins. In the definition of Emp\_SIGMA we define the columns of the PRIOR\_SOURCE data file to use as the basis of the empirical sampling distribution of SIGMA.

In the INPUT\_PRIOR\_DATA block we specify how the defined PRIOR\_SOURCE information is to be mapped to vectors and matrices defined in the PRIOR\_VARIABLE\_DEFINITION block and used in the definition of the sampling distributions. Note that in the PRIOR\_VARIABLE\_DEFINITION block we must define the type of the objects p\_k\_v, bins\_k\_v and data\_SIGMA2 (vector, matrix and vector respectively). POP\_k\_v is then sampled from a multivariate non-parametric sampling distribution with bins specified by the matrix bins\_k\_v and sampling probabilities by the vector p\_k\_v, while POP\_SIGMA2 is sampled from an empirical sampling distribution with values held in the vector data\_SIGMA2.
7 The Design Object

7.1 Design Object overview

The Design Object is intended to provide information for creating simulation or evaluation of trial designs, and design space information required for optimal design. For trial simulation or design evaluation, the trial design is fixed and should be completely defined. For optimal design the user should specify a design space to optimise over in the DESIGN_SPACES block.

The user should specify all required interventions (drug administrations), sampling (or observation) schedules, populations, covariate distributions, and design parameters. These are then combined in the STUDY_DESIGN block to completely define study design(s). For simulation and evaluation only one design should be specified. For optimal design, many study designs can be considered and the DESIGN_SPACES block will determine what attributes the optimisation algorithm will search over.

An elementary design is given by the combination of INTERVENTION + SAMPLING, and can describe the design for groups of subjects e.g. treatment arms, or can define a unique design for an individual. In optimal design the combination of treatment arms and numbers of subjects in each is called the population design.

7.2 DECLARED_VARIABLES block

As with the DECLARED_VARIABLES block in other MDL objects, the block should contain any variables defined in other MDL Objects (typically the Model Object) that are required for complete definition of the Design Object. See section Error! Reference source not found. for further details.

7.3 INTERVENTION block

The INTERVENTION block contains details of administration schedules and types and how these are combined to form treatment interventions. Note here that we distinguish between treatment interventions (including placebo treatment, non-medical treatments, drug-free periods) and treatment arms, since an arm of a study may be purely observational i.e. with no treatment intervention. For the optimisation task of optimal design, the INTERVENTION block defines the treatment definitions for the starting design (if required by the algorithm).

The syntax for this block is:

```plaintext
<name> : { type is bolus,
    input = <Model Object dosingTarget>,
    amount = <real value or vector of real values>,
    doseTime = <vector of real values>,
    ssInterval = <real value>,
    timeLastSSDose = <real value>,
    doseIntervalVar = <dosing interval variable reference>,
    lastDoseTimeVar = <time of last dose variable reference>
}
```

In the above, ssInterval and timeLastSSDose are optional; doseIntervalVar, lastDoseTimeVar are optional.

```plaintext
<name> : { type is infusion,
    input = <Model Object dosingTarget>,
```


amount = <real value>,
rate = <vector of real values>,
doseTime = <vector of real values>,
duration = <vector of real values>,
ssInterval = <real value>,
timeLastSSDose = <real value>,
timeStopSSInfusion = <real value>,
doseIntervalVar = <dosing interval variable reference>,
lastDoseTimeVar = <time of last dose variable reference>
}

In the above, one of duration or rate must be present. If doseTime is not present, then timeLastSSDose or timeStopSSInfusion must be present.

{name} : { type is combi,
    combination = <vector of previously defined interventions>,
    start = <vector of start times>,
    end = <vector of end times>
}

The “combi” type is used to combine existing (already defined) intervention schedules to form more complex dosing regimens.

{name} : { type is reset,
    reset = { variable = <Model Object dosingTarget variable>,
        resetTime=<real value>,
        value=<real value>
}

The “reset” type is used to reset the dosing variable in the Model Object at a given time.

{name} : { type is resetAll}

The “resetAll” type is used to reset all dosing variables.

For example, defining a single oral administration to the GUT at time 0 for use with the warfarin example (/Design/UseCase1_design1_eval):

```
INTERVENTION{
    admin1 : {type is bolus, input=GUT, amount=100, doseTime=[0] }
}
```

Another example showing single dose administration with an oral dose to the GUT and an infusion to the CENTRAL compartment (UseCase4_design1_eval):

```
INTERVENTION{
    admin1 : {type is infusion, input=CENTRAL, amount=100, doseTime=[0], duration=100 }  
    admin2 : {type is bolus, input=GUT, amount=150, doseTime=[0] }
}
```
A more complex example showing how interventions can be combined to define sequences of interventions and actions.

**INTERVENTION**

```
ivbolus : { type is bolus, input=CENTRAL, amount=100, doseTime=0 }
oral4 : { type is bolus, input=GUT, amount=150, doseTime=[0, 24, 48, 72] }
ivinf : { type is infusion, input=CENTRAL, amount=100, doseTime=0, duration=1 }
wash1 : { type is resetAll }
ivoral : { type is combi, combination = [ivbolus, wash1, oral4], start=[0, 24, 48] }
oralinf : { type is combi, combination = [oral4, wash1, ivinf], start=[0, 96, 120] }
```

The above example describes three different drug administrations - an IV bolus dose, four doses of oral drug separated by 24 hours, and an IV infusion over the space of 1 hour. It also describes a washout “event” which resets the amount in all dosing variables in the model. We can then combine these four interventions and actions into a treatment arm definition. So the ivoral arm receives the ivbolus dose at time zero, then a washout / reset at 24 hours followed by the four oral doses starting at 48 hours. Note that the doseTime argument in each intervention definition defines the dosing time relative to the respective value in the start argument of the combination definition. Similarly, the oralinf arm receives the four doses of oral drug, followed by a washout / reset event at 96 hours then the ivinf treatment starting at 120 hours.

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>24</th>
<th>48</th>
<th>72</th>
<th>96</th>
<th>120</th>
</tr>
</thead>
</table>

Note that for vectors of times, the seq(...) function can be used to specify a sequence of times with starting time, stopping time and interval size (MDL_Reference Guide section 5.80).

### 7.4 SAMPLING block

Similar to the INTERVENTION block, the SAMPLING block provides a means to describe observation schedules which can then be used for individuals, for treatment arms or in combination to define complex patterns of observation. For the optimisation task in optimal design, the SAMPLING block defines the starting design sample times (if required by the algorithm).

The syntax for this block is:

```
<name> : { type is simple,
              outcome = <Model Object OBSERVATION block variable>,
              sampleTime = <vector of real values>,
              numberTimes = <integer>,
              deltaTime = <real value>,
              blq = <real value>,
              ulq = <real value>,
          }
```

In the above definition, the outcome argument is mandatory, all other arguments are optional. numberTimes defaults to the length of the sampleTime vector. Either numberTimes or sampleTime
should be used, not both. Note that `numberTimes` should only be used in the context of Optimal Design optimisation tasks and not for design evaluation or simulation. `deltaTime` is the minimum time between two samples (for use in optimal design).

To combine a number of sampling definitions, the user can use the following syntax:

```plaintext
<name> { type is combi,
    combination = [<vector of previously defined sampling schedules>,
    numberTimes = <integer>,
    start = <vector of real values>,
    relative = <Boolean>
}
```

The combination argument is mandatory, other arguments are optional.

For an example of definition of “simple” sampling scheme (UseCase1_design1_eval):

```plaintext
SAMPLING{
    window1 : { type is simple, outcome=Y,
                 sampleTime = [0.0001, 24, 36, 48, 72, 96, 120] }
}
```

For an example of combining sampling schedules (UseCase3_design1_eval):

```plaintext
SAMPLING{
    winPK : { type is simple, outcome=CC,
             sampleTime = [0.0001, 24, 36, 48, 72, 96, 120] }
    winPD : { type is simple, outcome=PCA,
             sampleTime = [0.0001, 24, 36, 48, 72, 96, 120] }
    # implies concurrent start, both start 0 unless define times
    sampPKPD : { type is combi, combination=[winPK,winPD] }
}
```

In the above, the sampling schedules for winPK and winPD start concurrently (at time 0) - there is no vector of `start` times to offset winPK and winPD.

### 7.5 POPULATION block

The `POPULATION` block initialises and defines population characteristics or distributions for use in defining covariate distributions within the Model Object `COVARIATES` block. The `POPULATION` block acts like a `COVARIATES` block for the Design Object, where the user defines covariate distributions and how these may differ for each study arm. The `COVARIATES` block in the Model Object defines how (observed) covariates are used in the model.

The syntax for the `POPULATION` block is:

```plaintext
<population_name> : { type is template,
    covariate = [<list of covariate definitions>] }
```

Each covariate definition should have the following syntax. For continuous covariates:

```plaintext
{cov = <covariate name>, rv ~ <ProbOnto distribution>}
```

For discrete covariates:

```plaintext
{catCov = <covariate name>, discreteRv ~ <ProbOnto distribution>}
```
It is possible to define an arm to contain only one level of a categorical covariate (defined in the `POPULATION` block) by using the following syntax:

```plaintext
{catCovValue = <categorical covariate category>}
```

Note that covariates defined in the `POPULATION` block should be declared in the `DECLARED_VARIABLE` block.

For example, for the Design Object to be used with UseCase5 which uses WT, SEX and AGE as covariates (UseCase5_design2_opt.mdl):

```plaintext
DECLARED_VARIABLES{
    ...
    SEX withCategories{female, male}
}

POPULATION{
    default : { type is template, covariate = { catCov=SEX,
        discreteRv ~ Bernoulli1(probability = 0.5) } }
    arm2Pop : { type is template, covariate = { catCovValue=SEX.female } }
}
```

In the above example, a default distribution is defined for the SEX covariate which has categories female and male and defines that 50% of subjects should be male. (The Bernoulli distribution defines probability of the second category). arm2Pop is defined as having ONLY female subjects.

Another example for the same model:

```plaintext
POPULATION{
    default : { type is template, covariate=[{ catCov=SEX, discreteRv ~ Bernoulli1(probability = 0.5) },
        { cov = WT, rv ~ Normal1(mean= piecewise{{ 70 when SEX==SEX.male; otherwise 60 }},
          stddev=10) }
    }
    arm2Pop : { type is template, covariate=[{ catCovValue=SEX.female },
        { cov = WT, rv ~ Normal1(mean=55, stddev=5) }]
    }
}
```

In the above example, SEX is defined as having 50% probability of being male, and WT is defined as having a Normal1 distribution where the mean depends on the value of SEX. If SEX is male then the mean is 70, whereas if female then the mean is 60. Both populations have a standard deviation of 10 for WT. arm2Pop is exclusively female and WT is defined separately for this group.

The populations defined in the `POPULATION` block are used in definition of the study arms within the `STUDY_DESIGN` block. For example:

```plaintext
STUDY_DESIGN{
    arm1 : { armSize=16,
        interventionSequence=[[{ admin=admin1, start=0 }]],
        samplingSequence=[[ ...
```
7.6 STUDY DESIGN block

The STUDY DESIGN block defines how the SAMPLING, INTERVENTION and POPULATION blocks are used to define arms in a study design. For the optimisation task of optimal design, it defines the starting design (if required by the algorithm). For evaluation and simulation tasks it defines the study design to be evaluated or simulated.

The syntax for defining each arm is as follows:

<name> : {
  armSize = <integer value>,
  sameTimes = <Boolean>,
  occasionSequence = [{ occasion=<vector>, level=<reference varLevel>, start=<vector> }],
  interventionSequence = [{ admin = <INTERVENTION block list name>, start = <real value> }],
  samplingSequence = [{ sample = <SAMPLING block list name>, start = <real value> }],
  population = <POPULATION block list name>
}

Recall that the intervention (doseTime argument) and sampling times (sampleTime argument) specified are relative. Within the STUDY DESIGN block definitions we can specify start times for these that define in study time, when each starts. Using this construct it is possible to define interventions and samples that happen concurrently, and also those that happen sequentially. If the start argument is not given in interventionSequence or samplingSequence then it is assumed to be zero and the start times are taken from the relevant INTERVENTION and SAMPLING block definitions.

armSize defines the size of each arm.

If multiple outcomes are defined, then sameTimes (if true) denotes that the same observation times are to be used for all outcomes.

Additionally and optionally, arguments may be given pertaining to the study design as a whole. These are defined using the set keyword and are comma separated. Arguments used are:

- totalSize = <integer value>
- numberSamples = <vector of integer>
- totalCost = <real value>
- numberArms = <vector of integer>
- sameTimes = <Boolean>
**totalSize** defines the overall size of the study. This defaults to the sum of the individual armSize arguments.

**numberSamples** defines the constrain on the number of samples for one subject. If one value is given, the number of samples should be equal to that value for all designs. If several values are given then this defines the set of number of samples allowable in designs (primarily used in optimal design).

**totalCost** defines the total cost for the entire population design.

**numberArms** defines the constraint on the number of arms in the study. If a single value is given then the final design will have exactly the required number of arms. If several values are given then this defines the set of number of arms allowable in designs (primarily used in optimal design).

**sameTimes** defines whether the sample sampling times are to be used for all observed outcomes.

As an example of the STUDY_DESIGN block (UseCase5_design2_opt.mdl):

```plaintext
STUDY_DESIGN{
    arm1 : {
        armSize=16,
        interventionSequence=[{
            admin=admin1,
            start=0
        }],
        samplingSequence=[{
            sample=window1,
            start=0
        }],
        population = default
    }

    arm2 : {
        armSize=17,
        interventionSequence=[{
            admin=admin1,
            start=0
        }],
        samplingSequence=[{
            sample=window1,
            start=0
        }],
        population = arm2Pop
    }
}
```

Another example showing how arguments like totalSize and numberSamples should be used (when optimising a study design):

```plaintext
STUDY_DESIGN{
    set totalSize=40,
    numberSamples=4
    arm1 : {
        interventionSequence=[{
            admin=admin1,
            start=0
        }],
        samplingSequence=[{
```
7.7 DESIGN_SPACES block

The DESIGN_SPACES block defines which design elements should be optimised in finding the optimal design. A design space defines the possible values of the design variables specified in SAMPLING, INTERVENTION, POPULATION blocks. It should not be used with design evaluation (EVALUATE task in the Task Properties object) or simulation. If no design space is defined for a variable e.g. amount or sampling schedule then it is assumed fixed for the optimisation process.

The syntax for DESIGN_SPACES definitions is as follows:

```
<name> : {
    objRef = <named object in INTERVENTION, SAMPLING or POPULATION>,
    element is <variable from named object above>,
    discrete = <vector of values, with type dependent on original argument >,
    range = <vector of values defining upper and lower range of values to be explored,
    with type dependent on original argument>
}
```

Valid choices for element depend on the object type (SAMPLING, INTERVENTION, POPULATION) that is being referenced: bolusAmt, infAmt, duration, sampleTime, numberTimes, covariate, numberArms, armSize, parameter, doseTime.

For example (UseCase1_design2_optFW.mdl):

```
warfarin_PK_ODE_design = designObj{
    ...
    SAMPLING{
        window1 : { type is simple, sampleTime = [0.0001, 36, 96, 120], outcome=Y }
    }
    DESIGN_SPACES{
        DS1 : { objRef=[window1], element is sampleTime,
                discrete=[0.0001,24, 36,48,72,96, 120] }
        DS2 : { objRef=[window1], element is numberTimes,
                discrete=[4,5] }
    }
    ...
}
```

In the above example the DESIGN_SPACES defines how the optimisation algorithm will investigate the optimal design based on the window1 defined within the SAMPLING block, examining designs with 4 or 5 sample points chosen from the sample times defined in DS1 i.e. [0.0001, 24, 36, 48, 72, 96, 120]. The sample times defined in window1 of the SAMPLING block are used as the starting design for optimisation.

Another example:
warfarin_PK_SEX_design = designObj{
    ...  
    INTERVENTION{
        admin1 : {type is bolus, input=INPUT_KA, amount=100, doseTime=[0] }
    }
    SAMPLING{
        window1 : {type is simple, sampleTime = [0.0001, 24, 36, 48, 72, 96, 120], outcome=Y }
    }
    STUDY_DESIGN{
        arm1 : {
            armSize=16,
            interventionSequence=[[admin=admin1, start=0]],
            samplingSequence=[[sample=window1, start=0]]
        },
        arm2 : {
            armSize=17,
            interventionSequence=[[admin=admin1, start=0]],
            samplingSequence=[[sample=window1, start=0]]
        }
    }
}

DESIGN_SPACES{
    DS1 : { objRef=[window1], element is sampleTime, discrete=[0.0001, 24, 36, 48, 72, 96, 120] }
    DS2 : { objRef=[window1], element is numberTimes, discrete=[4,5] }
    DS3 : { objRef=[admin1], element is bolusAmt, discrete=seq(100,300,100) }
    DS4 : { objRef=[arm1, arm2], element is armSize, range=[0,30] }
}

In the above example several design space elements are defined. DS1 defines possible sampling times for window1, while DS2 defines that designs of 4 or 5 samples are to be considered. DS3 specifies that the amount to be dosed should vary between 100 and 300 with steps of 100. Finally DS4 specifies how the number of subjects in each arm should be in the range [0,30] (inclusive) for both arms.

### 7.8 DESIGN_PARAMETERS block

This block defines parameter values (constants) required for use in defining the design or to pass as constants to the model. Values are defined by assignment `<name> = <value>` or by equation `<name> = <equation>`.

Variables defined in DESIGN_PARAMETERS can be used in definition of covariate distributions defined in Design Object blocks.
Recall the example above where the distribution of covariates was defined in the `POPULATION` block:

```
POPULATION{
  default : { type is template, 
    covariate=[[ { catCov=SEX, discreteRv ~ Bernoulli1(probability = 0.5) }, 
                   { cov = WT, rv ~ Normal1(mean= 
                       piecewise{{ 70 when SEX==SEX.male;
                               otherwise 60 }}, 
                           stdev=10) } ]
     ]
  }
  arm2Pop : { type is template, 
    covariate=[[ { catCovValue=SEX.female },
                   { cov = WT, rv ~ Normal1(mean=55, stdev=5) } ]
     ]
  }
}
```

We can define the population means for weight in each population in the `DESIGN_PARAMETERS` block and pass these values into the `POPULATION` block definitions.

```
DESIGN_PARAMETERS{ 
  maleMeanWT = 70 
  femaleMeanWT = 60 
  femaleMeanWTArm2 = 55 
  stdevWT = 10 
}

POPULATION{
  default : { type is template, 
    covariate=[[ { catCov=SEX, discreteRv ~ Bernoulli1(probability = 0.5) },
                   { cov = WT, rv ~ Normal1(mean= 
                       piecewise{{ maleMeanWT when SEX==SEX.male;
                               otherwise femaleMeanWT }},
                           stdev=stdevWT) } ]
     ]
  }
  arm2Pop : { type is template, 
    covariate=[[ { catCovValue=SEX.female },
                   { cov = WT, rv ~ Normal1(mean=femaleMeanWTArm2, stdev=stdevWT) } ]
     ]
  }
}
```
8 The MOG Object

The MOG Object is where the user defines the MDL objects required for a particular task: estimation, simulation, design evaluation or optimisation.

8.1 INFO Block

The INFO block provides a name and/or problem statement to the associated MOG. The name attribute populates the Name tag in PharmML, while the problemStmt attribute populates the Description tag in PharmML. This information can then be passed forward to target software that support names or problem statement definition. For example, NONMEM conversion uses the problemStmt attribute to populate the $PROB statement, while the name attribute is converted to metadata in the comment header of the control stream file.

By default the Name tag in PharmML is “Generated from MDL. MOG ID: <MOG object name>”.

The problemStmt attribute can be set via the ddmore R package function writeMDL( ... , problemStmt = “Problem statement text”).

The syntax for the INFO block is:

```
INFO{ 
  set problemStmt = <text string> , 
  name = <text string> 
}
```

The statements can be comma separated or the set command can be used for each line. For example (UseCase1_1.mdl):

```
INFO{ 
  set problemStmt = "my Problem Statement" 
  set name = "10May2016 Task Properties check" 
}
```

8.2 OBJECTS Block

In the current MDL version, the only block supported is the OBJECTS block.

The OBJECTS block defines the objects (defined in the current .mdl file) that are to be used in defining the Modelling Objects Group for use in the desired task. The MDL-IDE checks that these named objects exist in the current file.

The MDL-IDE also uses the MOG Object to “tie together” variable definitions across objects - it checks that variables used in the model are defined. So for example, if the model expects a covariate called logtWT but this is not defined in the Data Object then an error is given. Without a MOG Object, no validation check of this type is possible. Without the MOG Object, the MDL-IDE can only perform rudimentary syntax checking of MDL statements. With the MOG Object defined the MDL-IDE can check that the resulting model will result in valid PharmML.

The syntax for statements in this block is:

```
<Object name within the current MDL file> : {type is <dataObj | designObj | mdlObj | parObj | priorObj | taskObj>}
```

Note that in the MOG Object, the user must specify dataobj OR designObj; parObj OR priorObj. As stated previously, for simulation, design evaluation or optimisation the Design Object takes the place of the Data Object. Similarly for estimation with BUGS or other Bayesian estimation software the Prior Object takes the place of the Parameter Object.

For example (UseCase1.mdl):
warfarin_PK_ODE_mog = mogObj {
  OBJECTS{
    warfarin_PK_ODE_dat : { type is dataObj }
    warfarin_PK_ODE_mdl : { type is mdlObj }
    warfarin_PK_ODE_par : { type is parObj }
    warfarin_PK_ODE_task : { type is taskObj }
  }
}

8.3 Mapping of variable names between MDL Objects

The current version of MDL requires that variable names in each object are consistently named. Future versions of MDL may allow mapping between variable names across objects.
9 MDL Language Reference

The aim of this section is to provide the technical aspects behind the MDL language: documenting its syntax and semantics in detail. New users may wish to read the sections on Data Object, Parameter Object, Model Object, Task Properties Object and Model Object Group, and explore the MDL implemented in the Use Case examples first in order to familiarise themselves with how MDL is used to define models. The information in this section may be of more interest to users who are writing their own models using MDL and wish to know the detail of syntax and grammar implemented in the MDL-IDE.

Like many computational languages MDL has two layers. First is the syntactic layer, or core, that defines how the words and symbols of the language are combined together in meaningful ways. This is like the building blocks of the language, its vocabulary, punctuation and grammar. Building on this foundation then is the second, semantic layer of MDL. This is where the meaning of the language is defined and how the building blocks of the core are used to create a language that describes pharmacometric models.

This organisation is reflected in this section, which starts with the description of the language core, followed by an explanation of MDL’s type system before moving on to the semantics layer of MDL.

9.1 Core syntactic elements

The core units of the language are described here from the bottom up. Starting with the language keywords, through the definitions of expressions and statements until we reach the highest level of organisation in MDL, the object.

9.1.1 Keywords

The keyword names are reserved and cannot be reused elsewhere, for example as attribute or variable names. The keywords in MDL have deliberately been kept to a minimum and at present there are 16. They are:

as, if, else, elseif, ln, exponential, false, in, inf, is, otherwise, pi, piecewise, set, then, when, withCategories, true

include a keyword that are not currently used, but which is reserved for future versions of MDL:

ordered

9.1.2 Variable names

Variables names in MDL must conform to the following rules:

- There are no reserved variable names in MDL
- Variable names may only contain letters or numbers and ‘’ and must start with a letter or ‘’ character.
- As MDL is a case sensitive language the case of letters matters in variable names so ‘t’ is a different variable to ‘T’.

In technical terms a variable name must comply with the following regular expression:

(`a'..'z'|'A'..'Z'|'_'('a'..'z'|'A'..'Z'|'_'|'0'..'9')*)

An addition constraint not reflected in the above regular expression is that the variable name also cannot begin with ‘MDL__’. This prefix is reserved for internal used by code generators that may be used to convert MDL to other languages and may need to create synthetic variable names.

9.1.3 Literals

Values such as numbers and strings etc can be written explicitly in MDL. Technically such values are referred to as literals. MDL supports the following types:
- **Vector:** [0, 1, 2, 25.0]
- **Matrix:** [[0, 20, 2; 23.5, 20, 1]]
- **Strings:** “a string”
- **Integers:** 99, 22, 0, -1, -477
- **Real:** 99.9, -0.473, 9e-2, -0.3424e5
- **Boolean:** true, false

Note that in the current MDL a vector containing only integer values will be regarded as a vector of type integer (not of type real). To ensure that the vector is of type real, the user may need to specify one of the numbers as a real value e.g. 25.0.

### 9.1.4 Expressions

An expression in MDL is primarily used to express mathematical concepts and evaluate mathematics. Expressions are divided into two types, Boolean and numerical. The former is an expression that evaluates to a Boolean (True or False), while the latter evaluates to a Real number. Examples are:

\[
\begin{align*}
&x > 5 \land y \leq 0 \\
&x - 5 \times 23 \\
&x^{(2 \times y/z)}
\end{align*}
\]

Expressions can contain mathematical functions too:

\[
\begin{align*}
&\sin(x) \\
&\ln(x + y) + \ln(22)
\end{align*}
\]

Some functions can use named arguments:

\[
x + \text{func}(\text{arg1}=1, \text{arg2}=3)
\]

In MDL conditional statements are also expressions:

\[
x \times \text{if}(y > 22) \text{ then } 300 \times a \text{ else } 1
\]

And they can use categorical variables:

\[
x \times \text{if}(\text{sex} == \text{sex.female}) \text{ then } 1 \text{ else } 0
\]

### 9.1.4.1 Numerical and Boolean Expression

Expressions are built up using operators that either take one or two operands:

- binary_op := <operand> <operator> <operand>
- unary_op := <operator> <operand>

Logical expressions are formed using combinations of Boolean operators (\&\&, ||, !) or comparison operators (<, >, <=, >=, !=, ==). Numerical expressions use the standard mathematical operators (+, -, /, *, ^, \%). These are shown below.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Symbol</th>
<th>Left Type</th>
<th>Right Type</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logical AND</td>
<td>&amp;&amp;</td>
<td>Boolean</td>
<td>Boolean</td>
<td>Boolean</td>
</tr>
<tr>
<td>Logical OR</td>
<td>Boolean</td>
<td>Boolean</td>
<td>Boolean</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>Less than</td>
<td>&lt;</td>
<td>Real</td>
<td>Real</td>
<td>Boolean</td>
</tr>
<tr>
<td>Greater than</td>
<td>&gt;</td>
<td>Real</td>
<td>Real</td>
<td>Boolean</td>
</tr>
<tr>
<td>Less than or equal</td>
<td>&lt;=</td>
<td>Real</td>
<td>Real</td>
<td>Boolean</td>
</tr>
<tr>
<td>Greater than or equal</td>
<td>&gt;=</td>
<td>Real</td>
<td>Real</td>
<td>Boolean</td>
</tr>
<tr>
<td>Equal</td>
<td>==</td>
<td>Real</td>
<td>Real</td>
<td>Boolean</td>
</tr>
<tr>
<td>Not equal</td>
<td>!=</td>
<td>Real</td>
<td>Real</td>
<td>Boolean</td>
</tr>
<tr>
<td>Power</td>
<td>^</td>
<td>Real</td>
<td>Real</td>
<td>Real</td>
</tr>
<tr>
<td>Multiplied by</td>
<td>*</td>
<td>Real</td>
<td>Real</td>
<td>Real</td>
</tr>
<tr>
<td>Divided by</td>
<td>/</td>
<td>Real</td>
<td>Real</td>
<td>Real</td>
</tr>
<tr>
<td>Modulo (remainder)</td>
<td>%</td>
<td>Real</td>
<td>Real</td>
<td>Real</td>
</tr>
<tr>
<td>Add</td>
<td>+</td>
<td>Real</td>
<td>Real</td>
<td>Real</td>
</tr>
<tr>
<td>Subtract</td>
<td>-</td>
<td>Real</td>
<td>Real</td>
<td>Real</td>
</tr>
<tr>
<td>Negation (unary)</td>
<td>-</td>
<td>Real</td>
<td>Real</td>
<td>Real</td>
</tr>
<tr>
<td>Positive (unary)</td>
<td></td>
<td>Real</td>
<td>Real</td>
<td>Real</td>
</tr>
</tbody>
</table>

The operators have the same operator precedence you would expect in a standard mathematical equation. In the table below operator precedence is shown, ordered from highest to lowest.

<table>
<thead>
<tr>
<th>Operators</th>
<th>Precedence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unary</td>
<td>+ - !</td>
</tr>
<tr>
<td>Power</td>
<td>^</td>
</tr>
<tr>
<td>Multiplicative</td>
<td>* / %</td>
</tr>
<tr>
<td>Additive</td>
<td>+ -</td>
</tr>
<tr>
<td>Relational</td>
<td>&lt; &gt; &lt;= &gt;=</td>
</tr>
<tr>
<td>Equality</td>
<td>== !=</td>
</tr>
<tr>
<td>Logical AND</td>
<td>&amp;&amp;</td>
</tr>
</tbody>
</table>
9.1.4.2 Variable references

A symbol name used in an expression is treated as a reference to a symbol defined elsewhere in the same object.

\[
\begin{align*}
a &= 10 \\
b &= 10 + a
\end{align*}
\]

The above code snippet shows how the variable reference ‘a’ in the expression refers to the definition of variable “a” which is initialised to 10. This is intuitive as is the fact the expression evaluates to 20. However, references to categorical variables behave slightly differently. A reference to it takes two forms:

1. A reference to the variable itself, e.g. sex
2. Or a reference to a category value, e.g. sex.female

Note that the second form uses a qualified name based on a combination of the categorical variable and its value:

\[<\text{categorical variable}>.<\text{category value}>\]

The meaning of a reference to a category variable is self-evident, however, a reference to the categorical variable itself is less so. Consider the following:

\[
\begin{align*}
\text{sex withCategories } \{\text{male, female}\} \\
\text{sex} &= \text{sex.female}
\end{align*}
\]

In essence, the reference to the categorical variable ‘sex’ is referring to the category value held by ‘sex’. If ‘sex’ holds the value ‘sex.female’ then this expression evaluates to true. This enables us to write conditions expressions (see below) like this:

\[
\text{if}(\text{sex} == \text{sex.female}) \text{ then } 1 \text{ else } 0
\]

9.1.4.3 Type Specifications

As is described below MDL (see Type System) infers the type of parameter or variable where it can. However, when a variable is not initialised the author must tell MDL what type to expect. The type specification syntax enables this. Simple examples are below:

A::real
B::string
C:: int
D::pdf
E::Boolean

The basic syntax is ‘::’ followed by the type name. Vectors, and matrices require more complex handling:

\[
\begin{align*}
F::\text{vector}[\cdot:\cdot\text{int}] & \quad \# \text{ vector of ints} \\
G::\text{matrix}[\cdot:\cdot\text{string}] & \quad \# \text{ matrix of strings} \\
H::\text{matrix}[\cdot:\cdot\text{vector}[\text{boolean}]] & \quad \# \text{ matrix of vectors of booleans}
\end{align*}
\]

As you can see vector types need a type specification to define its element type as does the matrix. In order to reduce typing some type specifications can be omitted in which case the type is assumed to be Real:
A                  # type real
F::vector        # vector of reals
G::matrix        # matrix of real
H::matrix[[ ::vector ]] # matrix of vectors of reals

### 9.1.4.4 Conditional expressions

Sometimes it is useful for an expression to evaluate to different values depending on some arbitrary criteria. In a mathematical expression this is handled by a piecewise function:

\[
f(x) = \begin{cases} 
-1, & x < 0 \\
1, & \text{otherwise}
\end{cases}
\]

In MDL we have a directly equivalent piecewise construct:

```
piecewise {{ -1 when x < 0; otherwise 1 }}
```

This can take more than 1 condition as you would expect:

```
piecewise {{ -1 when x < 0; 1 when x > 0; otherwise 0 }}
```

\[
f(x) = \begin{cases} 
-1, & x < 0 \\
1, & x > 0 \\
0, & \text{otherwise}
\end{cases}
\]

Or just condition and no otherwise clause:

Expressions with more than one condition are possible:

```
piecewise {{ -1 when x < 0; 1 when x >= 0 }}
```

which is equivalent to:

\[
f(x) = \begin{cases} 
-1, & x < 0 \\
1, & x \geq 0
\end{cases}
\]

**Equation 1**

A conditional expression should cover all possible conditions in order to prevent the generation of an undefined value, which will result in a runtime error. MDL does not enforce this, but to help ensure this it requires the writer provide at least two clauses. Ideally the last clause will be an ‘otherwise’ as this guarantees all conditions are covered, but in cases such as **Equation 1** this clearly is not necessary so this is not mandatory.

A related rule is that the conditions should also not define overlapping domains. In other-words only one condition can be true for any given set of values. This is illustrated by the code snippet below which breaks this rule:

```
piecewise {{ -1 when x < 0; 1 when x < 2; otherwise 0 }} # bad conditions!
```

The first two conditions can be true if \( x = -1 \) for example, which makes the correct evaluation of this expression impossible (remember that the written order of the conditions is not meaningful). This last rule is very important, because the order that the conditions are evaluated cannot be guaranteed and so the result of the above expression may not be as expected. At the moment MDL does not check this so the author must ensure that all conditions are independent.

Complementary to the piecewise expression is the if/else expression. Note that this is an expression not a statement so it is evaluated as part of a mathematical expression and is not used to control which statements are evaluated, as it would be in an imperative language such as R. The syntax is as you would expect:
if(x < 0) then -1 elseif(x > 0) then 1 else 0
Here the order of evaluation is significant. So ‘x < 0′ is evaluated before ‘x > 0’. This means that the
above example in an if/else expression is unambiguous:
if(x < 0) then -1 elseif(x < 2) then 1 else 0
as the condition ‘x < 2′ will never be evaluated. If/else expressions can be nested too:
if(x < 0) then -1 else if(x > 0) then 1 else 0
which is functionally equivalent to the ‘elseif’ non-nested equivalent.

9.1.4.5 Conditional Lists
A variant of the condition expression is the conditional list. This allows the conditional assignment to a
variable of a list. For example:

A : if(B > A) then { type is linear, pop=POP_A, ranEff = ETA_A }
    else { type is general, grp=GRP_A, ranEff = ETA_A }

This assign one or other of the lists to A depending on the condition. This follows the same rules as for
conditional expression, but the list being assigned must have the same List Super-type (see Type
System, below). In the above example both lists have a super-type of ‘IndivAbstractList’ so the list
assignment ‘:’ is valid.

A piecewise variant is also permitted:

A : piecewise {{
    { type is linear, pop=POP_A, ranEff = ETA_A } when B > A;
    otherwise { type is general, grp=GRP_A, ranEff = ETA_A }
}}

9.1.4.6 Functions
Functions take two forms in MDL. Simple and named argument functions. The latter is equivalent to a
standard mathematical function such as sin(a). The argument order is significant and all arguments are
required. So for example:

logx(y, b)
defines a logarithm of y to base b. Swapping the arguments around would change the meaning
accordingly.

The second form as one might expect takes named arguments and consequently the order of arguments
is not important and some arguments are optional. For example:

foo(arg1 = val1, arg2 = val2, arg3 = val3)
foo(arg1 = val1, arg2 = val2)
call the same function, but ‘arg3’ can be omitted. Typically the function will use a default value, but
the exact behaviour is determined by the definition of the function. Arguments can be constrained so
that only specific combinations are permitted as below:

foo(arg1 = val1, arg2 = val2, arg3 = val3)
foo(arg1 = val1, arg4 = val4, arg5 = val5)
foo(arg1 = val1, arg4 = val4, arg2 = val2)    # invalid
foo(arg1 = val1, arg5 = val5)                # invalid
Here the combinations of arg2, arg3 and arg4 and arg5 are permitted, but combination such as arg2 and arg4 are not. Likewise, arg5 cannot be used unless arg4 is also provided. This gives a lot of flexibility in the parameterisation of functions.

9.1.4.7 Function references

When a function is invoked a reference to that function’s definition is invoked and the appropriate value returned. For example, in the expression:

```
a = ln(10)
```

the symbol ‘ln’ is a reference to the function defining the natural log.

However, in some circumstances it is desirable to refer to a function, but not to evaluate it immediately, for example when referring to a function to be used for interpolation of data. In these circumstances a ‘&’ precedes the function name and the function arguments are omitted. For example:

```
Obs : { use is dv, variable = Y, interp=&linear }
```

Here the ‘interp’ attribute is assigned a reference to the ‘linear’ function definition. This function can then be invoked whenever interpolation between two data-points is required.

9.1.4.8 Vectors

Vectors are defined within in square brackets and can take numbers, strings, Boolean, variable references, logical and numerical expressions. Vectors must contain items of consistent type. The exception is when the vector contains numbers and variable references of numerical types (see type system below for more information). Some examples are below:

```
[ a, b-1, 1.0 * 3^5, 2.0^3.5, inf ]
[ “str”, “a str too”, “a”, “str” ]
[ true, false, true == false ]
```

Note that all elements must be populated. So the following is not permitted:

```
[ 3, , 3 ]  # forbidden
```

Vectors can also be nested, but again the vectors must have the same type:

```
[ [ 22, 45 ], [ 67, 89 ] ]
```

Vectors may be assigned to a variable or parameter and then used in other expressions:

```
A = [ 1.1, 2.2, 3.3, 4.4 ]
B = A  # B is assigned the contents of variable A
B = A[1]  # B is a scalar and assigned the first value in A: 1.1
B = A[2:3]  # B is vector and assigned elements 2 and 3 in A: [2.2, 3.3]
```

This illustrates how vector indexing works. The first element is at position 1 and the last index is equal to the length of the vector. If a single index value is used in square brackets, then a single scalar value is returned. As can be seen, MDL permits ranges of values to be specified, using a ‘:’ operator. This range operator is only permitted for vector and matrix indexing and ensures that the indexed vector returns a vector. This has a useful side effect when a vector containing a single vector is required:

```
B = A[3:3]
```

This returns a vector containing just element 3 of vector A, i.e. [3.3].

Vector index ranges can be omitted too. This indicates that indexing should start at the beginning or the end of the vector. For example:
A[:3]  # first element to element 3
A[2:]  # element 2 to the last element
A[:]  # first to last elements, i.e., the complete vector

Note that the last index is equivalent to the vector reference:
B = A[:]
B = A  # equivalent to above

9.1.4.9 Matrices
Matrices are very similar in their semantics to vectors and are indicated syntactically by the double square bracket with each row terminated by a ‘;’:

M = [[ 1, 2, 3; 3, 4, 6 ]]

Like vectors all cells in a matrix literal must be populated and in addition all rows must have the same number of columns:

[[ a, b, c;
  d, e;     # forbidden - inconsistent num columns
  g, h, i ]]

[[ a, b, c;
  d, , f;   # forbidden - missing value
  g, h, i ]]

Matrix indexing is similar to vector indexing and uses the same square bracket operator. Instead, an index for the row and column are specified:

M = [[ 1, 2, 3; 4, 5, 6 ]]
N = M[2, 3]  # a scalar value of 6

The first index indicates the row number and the second the column. As with vectors indexing starts at 1. Range operators behave in the same way as vectors, but in this case they always return a matrix:

N = M[2, 2:3]  # returns matrix: [[5, 6]]
N = M[1:2, 2]  # returns matrix: [[2; 5]]
N = M[1:2, 1:2]  # returns matrix: [[1, 2; 4, 5]]
N = M[1:1, 2:2]  # returns matrix: [[2]]
N = M[, 1]  # returns matrix: [[1; 4]]
N = M[, 1,2]  # returns matrix: [[1, 2; 4, 5]]
N = M[2,]  # returns matrix: [[4, 5, 6]]
N = M[,]  # equivalent to N = M

9.1.4.10 Sublist Expressions
The sublist is a convenient way of grouping together related pieces of information together in an expression. In its basic form the sublist is a set of attributes combined together as below:

{ att1 = val1, att2 = val2, ... }
Sublists which have different attribute sets are regarded as different. In fact sublists are fully fledged
types in MDL (see section ) so sublists with different attributes sets are distinct types. This is illustrated
in the example below where the sublists correspond to sublist types a and b:

\[
\begin{align*}
\{ \text{att1} = \text{val1}, \text{att3} = \text{val3} \} & \quad \# \text{sublist a} \\
\{ \text{att2} = \text{val2}, \text{att4} = \text{val4} \} & \quad \# \text{sublist b}
\end{align*}
\]

This means that an argument in a function (see section for more detail on argument typing) can
require a particular sublist type. In this way type system can ensure that only the valid type system is
used. For example if an attribute ‘foo’ expects a sublist of type a, then the following validity is
enforced:

\[
\begin{align*}
\text{foo} = \{ \text{att1} = \text{val1}, \text{att3} = \text{val3} \} & \quad \# \text{sublist a - valid} \\
\text{foo} = \{ \text{att2} = \text{val2}, \text{att4} = \text{val4} \} & \quad \# \text{sublist b - invalid}
\end{align*}
\]

How is the type of a sublist determined? Very simply all sublists must have a unique set of attributes.
The MDL processor takes the combination of attributes written and matches them to a sublist in its
dictionary of sublist types.

Sublist can also restrict the set of permitted attributes, just like you can with named function
arguments (see section 9.1.6.3). The details of how this is carried out is described in detail below
(section 0). The sublist can be contained in a vector as well. For example:

\[
[ \\
\{ \text{att1} = \text{val1}, \text{att2} = \text{val2} \}, \\
\{ \text{att1} = \text{val3}, \text{att2} = \text{val4} \},
]
\]

A Sublist can be used by any attribute, function argument or property. Below is an example of its used
in a function. It shows how the sublist provides a convenient way to group together sets of covariate
and fixed effect parameters in a linear individual parameter definition:

\[
\ln(\text{CL}) = \text{linear}( \text{trans is ln}, \text{pop = POP_CL}, \text{fixEff = [} \\
\{\text{coeff = BETA_CL_WT, cov = logtWT}, \\
\{\text{coeff = POP_FCL_FEM, catCov = SEX.female }, \\
\{\text{coeff = BETA_CL_AGE, cov = tAGE} \\
\}], \\
\text{ranEff = [ETA_CL]} )
\]

### 9.1.4.11 Variable selection expression

MDL provides support for conditionally assigning values in a variable to another variable using a special
syntactic structure called a variable selection expression. This construct is often used in a list
(described in section ) and is best illustrated by an example:

\[
\text{CMT: \{ use is \text{cmt} \}} \\
\text{AMT: \{ use is \text{amt}, \text{define=\{1 in \text{CMT} as \text{GUT}, 2 in \text{CMT} as \text{CENTRAL}\}} \}
\]

The value mapping syntax is used with the ‘define’ attribute. It can be read as “if value is ‘1’ in
variable ‘CMT’ then select as variable ‘GUT’”, if value is 2 in ‘CMT’ then select as variable ‘CENTRAL’”.
The semantics of what selection means can vary depending on context. In the above example, which is
valid MDL, the lists AMT and CMT each define a column in a dataset with the AMT column values being
assigned to the variable selected by the corresponding value in the CMT column.

The syntax for the variable selection expression is as follows:
{ <test value> in <qry var ref> as <select var ref>,
    <test value> in <qry var ref> as <select var ref>, ... }

The query (qry) and selection (select) variable references cannot be the same, the same query variable
must be used throughout the expression and the test value must be a numerical value.

9.1.4.12 Category value selection expression

Related to the variable selection expression is the category value selection. This carries out a similar
function, but for category values, in this case a particular category value is selected when a given
expression is matched. For example:

{ sex.female when 0, sex.male when 1 }

shows how sex.female is selected when the value is 0, and sex.male when it is 1. Typically this
expression is used in conjunction with a variable selection expression where the value in a list is to be
mapped to a categorical variable. This is illustrated below where the values in a DV column definition
are mapped to either a variable or a set of category values:

DVID: { use is dvid }
DV: { use is dv,
    define={1 in CMT as GUT,
                  2 in CMT as { Outcome.dead when 0, Outcome.alive when 1}
    }
}

The basic syntax is:

{ <category.value> when <test value>,
    <category.value> when <test value>, ... }

The test value should be a numerical value and the category values belong to the same category.

9.1.5 Attributes, Arguments, Properties and Values

Attributes in lists, arguments in functions and properties all behave in the same way in MDL. For the
sake of simplicity this description will refer to attributes, but this should be understood as a synonym
for argument.

An attribute is simply an identifier that is associated with a value. That value can be of any valid type
and is usually assigned to the with the ‘=’ operator. For example:

att1 = 0
att2 = true
att3 = “string val”
att4 = { a = 0, b = 3}
    # sublist
att5 = [0, 2, 3]
att6 = [1.5, inf, x]
att7 = { 1 in CMT as foo }
    # a mapping
att8 = varRef

In addition an attribute can be assign a value from a controlled vocabulary of options, called a built-in
enumeration. Because the MDL parser needs help to distinguish these option names from a variable
reference it is necessary to use a different assignment operator. Therefore, we use the keyword ‘is’ to
indicate that an attribute has been assigned an option. For example:
A typical usage of a built-in enumeration is to define the key value of a list, for example:

c1 : { use is id }
c2 : { use is adm, variable = D }
c3 : { use is idv }

Any name can be used as an attribute name as long as it is not a language keyword and it is a valid variable name.

In the previous version of MDL an attribute expecting a vector type would have to be written thus:

A : { foo = [ z ] } # foo expectes a vector

even when there was only one element in the vector. To avoid the additional typing and to improve readability it can now be written as:

A : { foo = z }

The ‘foo’ attribute still expects a vector, so behind the scenes MDL converts this value to [ z ].

9.1.6 Statements

The statement is the core of MDL and comes in several forms. However, they all have the following characteristics:

1. A statement can be split over any number of lines. The parser detects the start and end of the statement based on its context.
2. Sometimes it is helpful to the user to indicate where a statement starts and ends in which case an optional ‘;’ character can be used. Note this is completely optional.

The different statement types are below.

9.1.6.1 Equation definition

This defines a variable using a notation equivalent to a mathematical equation. It can have three forms:

1. \( x = \text{<expression>} \)

   \( x = 2 + 5 / \ln(22) \)

2. \( \text{fn}(x) = \text{<expression>}, \) where \( x \) is transformed by a function

   \( \ln(x) = 2 + r \)

3. \( x, \) where the variable \( x \) is declared but not initialised.

   \( x \)

The symbol (parameter or variable) it defines always has a type of Real.

In example 2 above, a transformation was used on the variable \( x \). It is important to note that the variable \( x \) can be used later (without the transformation) and it is implied that a back-transformation will have been applied. The user need not explicitly back-transform the parameter in the MDL code.

9.1.6.2 Category definition

A category definition creates a variable that has two roles. First it groups together a set of categories that belong to this variable and second it holds a value that is one of these categories. Exactly what this means is explained below, but here is the syntax of the category definition.

\( X \text{ withCategories} \{ \text{cat1, cat2, ... catN} \} \)
The definition can create any number of categories but must have at least one. A simple example is:

    sex withCategories { male, female }

The category values are specific for each category variable so the following is permitted:

    sex1 withCategories { male, female }
    sex2 withCategories { male, female }

The categorical variable always has a type of Enum.

9.1.6.3 List Definition

The list is a way of associating attributes and values with a variable. In many ways it is similar to a class seen in an Object Oriented programming language. The list has the following syntax:

    lst : { keyAtt (=|is) <value>, att (=|is) <value>, … }

Note that a list holds a specific set of attributes. The exact set is determined by the key attribute used, it’s value and the block containing the list. This is illustrated below.

```
BLK1{ 
    lst1 : { key is val1, att2 = val2, att3 = val3 } 
    lst2 : { key is val2, att20 = val20, att3 = val3 } 
}
```

In the above example the attribute named ‘key’ is the key attribute in block BLK1. Note that a block can have only one key attribute. That means, as in this example, the value may be used to distinguish between lists. So when the key attribute has a value of val1 the list uses a different set of attributes compared to when the value is val2.

```
BLK2{ 
    lst3 : { key = keyVal, att2 = val2, att3 = val3 } 
    lst4 : { key = keyVal, att2 = val2, att3 = val3 } 
}
```

BLK2 by contrast is configured not to use the value of the key attribute so each list must use the same set of attributes. Note that the attribute names can be the same across lists and the same key attribute name can be used in different blocks. Attribute names cannot be repeated within a list however and the key attribute is always mandatory.

9.1.6.4 Anonymous lists

The anonymous list is a legitimate version of a list, but it does not define a named list variable. This is used where one wishes to group together a set of attributes that are related to each other, but when we do not want the list to be referred to elsewhere. Its rules and behaviour are identical to this of the list in every other respect. Its syntax is as follows:

```
:: { keyAtt (=|is) <value>, att (=|is) <value>, … }
```

Note the ‘::’ symbol which designates this as an anonymous list.
9.1.6.5 Random variable definitions

Random variables are defined using the ‘~’ assignment operator which is the common mathematical convention when relating a random variable to a probability distribution. In most respects the random variable definition behaves like a standard equation definition except that the expression on the right hand side of the ‘~’ must have a type of Pdf. Note that the variable can have a transformation function on the left hand side. This is illustrated in the examples below:

\[
\begin{align*}
\text{ETA_CL} & \sim \text{Normal}(\text{mean} = 0, \text{sd} = \text{PPV_CL}) \quad \# \text{valid} \\
\ln(\text{ETA_CL}) & \sim \text{Normal}(\text{mean} = 0, \text{sd} = \text{PPV_CL}) \quad \# \text{valid} \\
\text{ETA_CL} & \sim \text{PPV_CL} \quad \# \text{invalid}
\end{align*}
\]

The generic syntax description is as follows:

\[
<\text{ID}> \sim \text{<PDF expression>}
\]

9.1.6.6 Category Lists

It is possible to define a list that also defines a set of categories. This type of definition allows the writer to associate a category definition with other attributes and information. It also supports the ability to select a category based on an expression. This is illustrated in the example below:

\[
\text{SEX}: \{ \text{use is catCov withCategories \{ M when } 0, \text{F when } 1 \} \}
\]

where the list definition, SEX, can be treated as a categorical variable with categories ‘M’ and ‘F’. This list also defines a data column so the ‘when’ syntax indicates that the ‘M’ category value is assigned to SEX if the data value is 0 and the ‘F’ category value if 1. This provides a shorthand that allows us to both define the categories and their mapping. The syntax of a category list is:

\[
<\text{ID}>: \{ \text{<attName>} \text{is <builtinEnum> withCategories \{<cat value> when <selection expr>,<cat value> when <selection expr> \} [, <attName> [is|=} <cat value>, …]} \}
\]

9.1.6.7 Property Definitions

Sometimes it is desirable to define properties that we wish to associate with a whole object. Sometimes this is because we want to define default attribute values for statements with the block or to set properties that only need to be defined once and which do not need to be referred to in an expression. To support this MDL provides the property syntax as follows:

\[
\text{set <prop name} [-|is] \text{<prop value} [, <prop name} [-|is] \text{<prop value}, \ldots\}
\]

An example of this can be found in the task object where the properties of the task, such as the estimation algorithm are set in this way:

\[
\text{ESTIMATION}{
    \text{set algo is saem}
}
\]

The property name is specific to the block and is unique to the block, so repeating property definition is forbidden. So:

\[
\text{ESTIMATION}{
    \text{set algo is saem, algo is fcei \# invalid}
}
\]

or

\[
\text{ESTIMATION}{
    \text{set algo is saem}
}
\]
9.1.7 Blocks

The block is used to organise similar concepts and it can be configured to only contain certain types of statement. As we have seen above the block also provides the context for what list and property attributes are available for use. The generic syntax is:

```
blkName [(name=value)] { <statement> [;] <statement> [;] … }
```

In the above syntax description a statement may also be another block and so in this way sub blocks may be nested within each other. In MDL such nesting is limited to one level as can be seen in the example below:

```
MODEL_PREDICTION {
  DEQ{
    RATEIN = if(T >= TLAG) then GUT * KA else 0
    GUT : { deriv =(- RATEIN), init = 0, x0 = 0 }
    CENTRAL : { deriv =RATEIN - CL * CENTRAL / V }
  }
  CC = CENTRAL / V
}
```

where the DEQ block is used to contain the definition of differential equations.

Blocks constraint the statements they contain in the number of ways:

1. by type. Some blocks may only permit lists or equation definitions or combinations of statement types.
2. by count. Each block defines the minimum and maximum number of statements it can contain.
3. by sub-block. The block may permit no sub-blocks or sub-blocks with specific names.

9.1.8 Objects

The object is the highest level of syntactic organisation in MDL. It defines a container for a set of blocks and the variables defined inside them. In MDL the object has a specific purpose and its semantic are a combination of that purpose and the semantics of the blocks and statements it contains. Its generic syntax is below:

```
<ID> = <objName> { <block> [, <block>, … ] }
```

where the objName is an internal MDL identifier for the type of the object. The object's type is related to semantic purpose. Note that the object type controls what types of block it contains. For example in MDL an obj of type 'dataObj' cannot contain an 'IDV' block. A short example of a dataObj is given below:

```
warfarin_PK_ODE_dat = dataObj {
  DECLARED_VARIABLES{GUT Y}
  DATA_INPUT_VARIABLES {
    ID : { use is id }
  }
```
The Type System

One of the core mechanisms for ensuring correctness in MDL is its type system. In this section we explain what the different types are, and any rules associated with their correct usage.

9.2.1 The types

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Int</td>
<td>Integer</td>
</tr>
<tr>
<td>Real</td>
<td>Real number</td>
</tr>
<tr>
<td>Boolean</td>
<td>Boolean</td>
</tr>
<tr>
<td>Enum</td>
<td>An enumeration type. A namespace for categories. Enumeration types do not require quotation marks (compared to strings).</td>
</tr>
<tr>
<td>Enum value</td>
<td>A specific enumeration defined by an enumeration type and the value held by a variable of enumeration type.</td>
</tr>
<tr>
<td>Builtin Enum</td>
<td>Commonly abbreviated to BE, this is a predefined set of enumerated values that are part of the MDL definition. These are usually prefixed by the 'is' keyword and are often (but not exclusively) used in lists to indicate the key attribute.</td>
</tr>
<tr>
<td>List</td>
<td>A data structure that associates a set of attributes with a variable. See below for details.</td>
</tr>
<tr>
<td>List Super-Type</td>
<td>An abstract type that a list can inherit from.</td>
</tr>
<tr>
<td>Pdf</td>
<td>A Probability density function. Usually returned by a statistical distribution function.</td>
</tr>
<tr>
<td>Random Variable</td>
<td>A value that is a random variable and was obtained from a Probability Distribution. It can have subsidiary types of Real (continuous probability distributions), Int and Enum (discrete probability distributions).</td>
</tr>
<tr>
<td>String</td>
<td>A character string</td>
</tr>
</tbody>
</table>
A probability mass function type. Typically returned by discrete probability distribution functions.

This is the type of the data mapping syntax structure.

This is a sublist, essentially an attribute and value pair.

A one dimensional array of values of a single type.

A two dimensional matrix.

A reference to a value. The value can be of any type.

No type. This is used internally to indicate a validation error during type checking. This is not a valid type.

9.2.2 The default type

In MDL the default type is Real. In a standard equation or random variable statement the symbol defined on the LHS of the definition is always of Real type. Examples:

\[ A = \langle \text{expression} \rangle \]
\[ \ln(B) = \langle \text{expression} \rangle \]
\[ C \sim \langle \text{expression} \rangle \]
\[ D \]

9.2.3 Type promotion

MDL allows an integer type to be used in mathematical expression. It does this using type promotion, where the integer value is automatically converted to a real value. This gives the kind of behaviour that the reader would expect. For example:

\[ A = 22.55 + 1 \]
\[ A = 22.55 + 1.0 \]

are equivalent. Note that mathematical expressions always evaluate to a value with a Real type so:

\[ A = 2 \times 55 \]

is effectively evaluated as:

\[ A = 2.0 \times 55.0 \]

9.2.4 Typing of more complex types

9.2.4.1 Vector type

A vector can potentially have elements of any type and in general all its elements must be of the same type. We refer to a vector type as “vector of type X” or an “X vector”. For example, vector of type String or String vector.

The type promotion rules above also apply to vectors, which means that a Real vector can contain a mixture of integer and real values.

Note that when writing a vector literal (see above) the type is inferred from its content. This means that for a vector to be of type Real it must contain at least one Real value as can be seen here:

\[ [ \ 0, \ 2, \ 3, \ 4 \ ] \quad \# \ \text{Int vector} \]
\[ [ \ 0, \ 2.0, \ 3, \ 4 \ ] \quad \# \ \text{Real vector} \]
9.2.4.2 Matrix type
This behaves identically to the vector type. The type of the matrix is inferred from its contents in exactly the same way.

9.2.4.3 List type
The type of the list is defined by a combination of its
1. owning block
2. key attribute
3. key value
This is best illustrated in the example below. Here the ‘c1’ variable is of type ‘List:Idv’ and ‘c2’ of type ‘List:Amt’. They both belong to the same block, the key attribute is ‘use’ so the discriminating factor in determining their type is the values ‘idv’ and ‘amt’.

```plaintext
DATA_INPUT_VARIABLES {
  c1 : { use is idv , variable= GUT }
  c2 : { use is amt , variable= GUT }
}
```

```plaintext
DATA_DERIVED_VARIABLES {
  c3 : { use is doseTime, idvColumn=c1, amtColumn=c2 }
}
```

In the DATA_DERIVED_VARIABLES block ‘c3’ is of a different type again, but contains two attributes, `idvColumn` and `amtColumn` that expect references to variables of type ‘List:Idv’ and ‘List:Amt’ respectively. List types are very specific for referencing another column type (as below) would result in a typing error:

```plaintext
DATA_DERIVED_VARIABLES{
  c3 : { use is doseTime, idvColumn=c1, amtColumn=c2 } # invalid
  # type error
}
```

In some cases the list value is not required to define the type. In the example below the type `List:deriv` is specified by just the block, ‘MODEL_PREDICTION’, and the key attribute ‘deriv’. As a consequence ‘GUT’ and ‘CENTRAL’ both have the same type.

```plaintext
MODEL_PREDICTION {
  DEQ{
    RATEIN = if(T >= TLAG) then GUT * KA else 0
    GUT : { deriv =(- RATEIN) }
    CENTRAL : { deriv =(RATEIN - CL * CENTRAL / V) }
  }
}
```
Each List type can potentially be converted to one another type when necessary. This is particularly useful if the semantics of the list make it desirable to use the list variables in an mathematical expression. The above example shows just such a case. The semantic of List:deriv lists is to define a difference equation, with the list variable corresponding to the derivative variable. The List:deriv type has a conversion type of Real. This means that when used in contexts that expect a Real type the Type system uses the conversion type. This allows the example above to be valid despite the fact that list variables are used as real values. All list type can potentially have one conversion type, but it’s use is optional and if not define then the list cannot be converted.

In the latest version of MDL there is an alternative method of identifying a list. This is via the attribute name. This acts as the list key and must be unique across all list types associated with the same block. For example, we can differentiate between different parameters as follows:

```
A : { value = 0.2 }
A : { vectorValue = [0.2, 0.3] }
A : { matrixValue = [[ 0.2, 0.4; 0.5, 0.9 ]] }
```

The above lists each have different types identified by the attribute name used.

### 9.2.4.4 List super-types

Sometimes it is useful to group related lists and refer to them interchangeably. For example, observation variables use different list types to define different continuous error models:

```
Y1 : { type is combinedError1 … }  # list type combinedError1List
Y2 : { type is combinedError2 … }  # list type combinedError2List
```

However, they are clearly related and should be compatible with each other. This is where the list super type is useful. In the above case both list types share the same super type, ‘observation’. This enables another attribute to be assigned this super-type and it will be able to refer to either of these variables:

```
Z1 : { superTypeAtt = Y1 } # valid
Z2 : { superTypeAtt = Y2 } # valid
```

which would not be possible with normal list typing:

```
X1 : { listTypeAtt = Y1 }  # valid as expects type is combinedError1List
X2 : { listTypeAtt = Y2 }  # invalid as expects type combinedError1List
```

In the same way super-types enable conditional list assignments that involve different list types:

```
U : if (x > 0) then { type is combinedError1, … }
else { type is combinedError2, … }
```

here ‘U’ is allocated the type ‘observation’ because both lists share the same super-type. In this way MDL knows that these lists are compatible.

### 9.2.4.5 Built-in enumeration type

A built-in enumeration is essentially a controlled vocabulary that constrains the values that can be assigned to an attribute. Each built-in enumeration is different and consists of a set of strings. The enumeration is match if the name assigned to the attribute is one of the names held by its built-in
type. As an example if there is a built-in type ‘eg’ that permits the names ‘foo’ and ‘bar’ then the following cases are valid and invalid:

\[
\begin{align*}
\text{att1 is foo } & \quad \text{# valid} \\
\text{att1 is bar } & \quad \text{# valid} \\
\text{att1 is ugg } & \quad \text{# invalid} \\
\text{att1 = foo } & \quad \text{# invalid}
\end{align*}
\]

The last case is important to note. MDL only knows to expect a building enumeration if it is preceded by the ‘is’ keyword. In the last statement above the assignment symbol was used and in this case MDL would treat this as a reference to a variable.

### 9.2.4.6 Sublist type

Sublists are types and they are distinguished from each other by their attributes. This is best illustrated by an example:

\[
\text{ln(CL) = linear( trans is ln, pop = POP_CL, fixEff = [}
\begin{align*}
\{\text{coeff = BETA_CL_WT, cov = logtWT}\},
\{\text{coeff = POP_FCL_FEM, catCov = SEX.female }\},
\{\text{coeff = BETA_CL_AGE, cov = tAGE}\}
\end{align*}
\]
\]
\[
\text{ranEff = [ETA_CL] )}
\]

Above the attribute ‘fixEff’ expects a vector type of ‘Sublist:FixEff’. The type system looks at the available sublist types and identifies the one that has the same attribute names. Note that the same type can allow different attribute combinations as can be seen in the example. If no subtype can be identified then the sublist is given a type of Undefined which will result in a typing error. Identifying the correct subtype also then allows the attributes in the sublist itself to be type checked.

### 9.2.4.7 Enum and Enum Value types

Enumeration types are unusual in MDL in that the type is to some extent defined within MDL. Take the following definitions:

\[
\begin{align*}
\text{Gender1 withCategories } & \{ \text{male, female, other } \} \\
\text{Gender2 withCategories } & \{ \text{male, female, other } \} \\
\text{State withCategories } & \{ \text{alive, dead } \}
\end{align*}
\]

The first defines an enumeration type with individual values, ‘male’, ‘female’ and ‘other’. The second definition is distinct from the first because it is associated with a different symbol. So in a sense the above definitions have created 3 new types called Gender1, Gender2 and State. However, the above statements also define 3 variables of the same name and as variables they can be initialised with one of their enumeration values. So the variable ‘State’ can hold a value of ‘alive’ or ‘dead’, or more correctly ‘State.alive’ or ‘State.dead’. This means that when used as a variable reference these variables always have a type of Enum value. This is illustrated by the example expressions below:

\[
\begin{align*}
\text{Gender1 == Gender1.male } & \quad \text{# valid} \\
\text{Gender2== Gender1.male } & \quad \text{# invalid} \\
\text{Gender1== Gender2 } & \quad \text{# invalid}
\end{align*}
\]
9.2.5 Type inference rules for Symbols

When a variable is defined it must be allocated a type and unlike language such as R, this type cannot change. Where is can MDL works out the type of the variable, usually from the type of any expression on the right hand side of an assignment operator (‘=’, ‘-’ or ‘:’). Where there is no assignment the variable type defaults to Real unless the author explicitly defines its type using a type specification (see above). This is best describe using examples:

A = [ 1, 2, 3 ]  # vector of ints
B            # real
C::string    # string
D = true     # Boolean
E ~ Poisson1(rate=lambda)  # Random Variable of int
F withCategories {heads, tails} ~ Bernoulli1(probability=p1)  # Random Variable of enum
G = [[ “A”, “B”; “C”, “D”]]  # matrix of strings

9.3 Scoping and statement ordering

MDL is declarative. It describes what the model is not how to implement it. In common with other declarative languages MDL has the following features:

- The order of blocks and statements within blocks is not significant.
- This means that symbols can be defined after they are referenced.
- A variable must not be assigned to more than once.
- In general a variable cannot be assigned to an expression that is dependent on itself. For example the following is not permitted:
  a = b
  b = c
  c = a (this makes a cycle back to the first statement)

A list can be defined that is exempt from this rule. For example, a derivative list can refer to itself:

x : { deriv = -x }

This is consistent with mathematical definitions such as:

dx/dt = -x

The scoping unit for MDL is the object. Variables defined inside an object are visible to expressions in the same object, but not outside it. This means that each MDL object is a self contained definition that does not rely on any other object.

9.4 MDL Reference Guide

Appendix X contains the MDL reference guide. This document gives detail of Object, Block, List, Sublist, Function and Type definitions. As it is based on and generated from the language definition file it documents exactly how the language is constructed and is definitive in describing what is valid MDL as implemented in the MDL-IDE. Elements of the Reference Guide have been extracted here in a format intended to help user understanding which may be used for quick reference.
9.4.1 How to read the MDL Reference Guide.
There is a hierarchy in the MDL Reference Guide. This reflects the way that users approach the
definition of a model in a sequential fashion. Understanding this hierarchy will help the user identify
what statements and attributes are valid as shown here:

- Object definitions describe which Blocks are valid within each Object.
- Block definitions describe
  - what arguments are associated with the block
  - what types of statements can be made within the block
  - what sub-blocks are permitted within the block
  - what list types are to be used with the block
  - what properties are permitted within the block.
- List types define
  - whether the list can be anonymous i.e. using ::{ type is ... }
  - whether the list can define categories
  - what types are associated with the list i.e. type is ...
  - what attributes are permitted within the list.
  - what the “signature” of the list looks like, including identifying which
    attributes are optional.
  - NB: list type names do not necessarily match the names used in MDL e.g. in
defining individual parameters we type “ CL : { type is linear, ... }” but the
type of the list in the Reference Guide is “IndivParamLinear”. However the list
type is given in the Block definition List table (left hand column) with the
associated MDL key value in the right hand column.
- Sublist definitions define the (small number of) cases where list types contain other
  lists, for example in definition of the fixed effect model within a IndivParamLinear
list. The sublist definition defines:
  - what attributes are permitted within the sublist
  - the permitted signature of the sublist and which attributes are optional.
- Function definitions define:
  - Arguments of the function
  - Type returned from the function.
- Type definitions define
  - the types and associated type classes.
  - the keyword types and associated enumeration types. e.g. type is
    combinedError1; set algo is saem.

9.5 Objects
Valid types of Objects are:
  dataObj, designObj, parObj, priorObj, mdlObj, taskObj, mogObj

9.6 Blocks
The following blocks are defined for the various MDL Objects:
<table>
<thead>
<tr>
<th>MDL Object</th>
<th>Block</th>
<th>Sub-blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>dataObj</td>
<td>DECLARED_VARIABLES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DATA_INPUT_VARIABLES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SOURCE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DATA_DERIVED_VARIABLES</td>
<td></td>
</tr>
<tr>
<td>designObj</td>
<td>DECLARED_VARIABLES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>INTERVENTION</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SAMPLING</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DESIGN_PARAMETERS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>POPULATION</td>
<td></td>
</tr>
<tr>
<td></td>
<td>STUDY_DESIGN</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DESIGN_SPACES</td>
<td></td>
</tr>
<tr>
<td>parObj</td>
<td>STRUCTURAL</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VARIABILITY</td>
<td></td>
</tr>
<tr>
<td>priorObj</td>
<td>PRIOR_PARAMETERS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NON_CANONICAL_DISTRIBUTION</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PRIOR_VARIABLE_DEFINITION</td>
<td></td>
</tr>
<tr>
<td>mdlObj</td>
<td>IDV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VARIABILITY_LEVELS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>COVARIATES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FUNCTIONS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>POPULATION_PARAMETERS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>STRUCTURAL_PARAMETERS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VARIABILITY_PARAMETERS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GROUP_VARIABLES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RANDOM_VARIABLE_DEFINITION</td>
<td></td>
</tr>
<tr>
<td></td>
<td>INDIVIDUAL_VARIABLES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MODEL_PREDICTION</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DEQ</td>
<td></td>
</tr>
<tr>
<td></td>
<td>COMPARTMENT</td>
<td></td>
</tr>
<tr>
<td>taskObj</td>
<td>ESTIMATE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SIMULATE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EVALUATE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>OPTIMISE</td>
<td></td>
</tr>
</tbody>
</table>
9.6.1 Functions
Mathematical functions are provided to support definition of models. The following mathematical functions are defined:

Statistical functions:
mean, median, probit, logit, invLogit, invProbit

Arithmetic functions:
log, log2, log10, ln, factorial, lnFactorial, floor, ceiling, min, max, abs, exp, sqrt, sum, toInt, seq, seqby, dseq, rep

Trigonometric functions:
sin, cos, tan, sinh, cosh, tanh, asin, acos, atan, asinh, acosh, atanh

vector and matrix handling:
toMatrixByRow, toMatrixByCol, asVector, inverse, triangle, transpose, diagonal, gInv, det, eigen, chol, matrix

Interpolation functions:
linearInterp, constInterp, lastValueInterp, nearestInterp, cubicInterp, pchipInterp, splineInterp

Distributions (see below).

9.6.2 Distributions
Distributions are denoted using the ~ prefix. The following distributions are defined:

<table>
<thead>
<tr>
<th>Name</th>
<th>Return Type</th>
<th>Argument name</th>
<th>Argument Types</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>Pdf</td>
<td>mean</td>
<td>Real</td>
<td>Maps to ProbOnto distribution Normal1</td>
</tr>
<tr>
<td>Normal</td>
<td>Pdf</td>
<td>mean var</td>
<td>Real</td>
<td>Maps to ProbOnto distribution Normal2</td>
</tr>
</tbody>
</table>

The following ProbOnto distributions are defined:

<table>
<thead>
<tr>
<th>Name</th>
<th>Return Type</th>
<th>Argument name</th>
<th>Argument Types</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal1</td>
<td>Pdf</td>
<td>mean stdev</td>
<td>Real</td>
<td>Note that ProbOnto argument is stdev not sd</td>
</tr>
<tr>
<td>Normal2</td>
<td>Pdf</td>
<td>mean var</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td>Normal3</td>
<td>Pdf</td>
<td>mean precision</td>
<td>Real</td>
<td>For use with BUGS</td>
</tr>
<tr>
<td>Name</td>
<td>Return Type</td>
<td>Argument name</td>
<td>Argument Types</td>
<td>Comment</td>
</tr>
<tr>
<td>--------------------</td>
<td>-------------</td>
<td>------------------------</td>
<td>----------------</td>
<td>---------------------------------------------------</td>
</tr>
<tr>
<td><strong>LogNormal1</strong></td>
<td>Pdf</td>
<td>meanLog</td>
<td>Real</td>
<td>Values given on the ln scale</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stdevLog</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>LogNormal2</strong></td>
<td>Pdf</td>
<td>meanLog</td>
<td>Real</td>
<td>Values given on the ln scale</td>
</tr>
<tr>
<td></td>
<td></td>
<td>varLog</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>LogNormal3</strong></td>
<td>Pdf</td>
<td>median</td>
<td>Real</td>
<td>median on the natural scale, standard deviation on the ln scale</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stdevLog</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>Bernoulli1</strong></td>
<td>Pmf</td>
<td>probability</td>
<td>Real</td>
<td>Probability argument specifies the probability of the second category.</td>
</tr>
<tr>
<td><strong>Poisson1</strong></td>
<td>Pmf</td>
<td>rate</td>
<td>Real</td>
<td>Probability of success (second category).</td>
</tr>
<tr>
<td><strong>Binomial1</strong></td>
<td>Pmf</td>
<td>probability</td>
<td>Real</td>
<td>Probability of success (second category).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>numberOfTrials</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>Beta1</strong></td>
<td>Pdf</td>
<td>alpha</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>beta</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>Gamma1</strong></td>
<td>Pdf</td>
<td>shape</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>scale</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>Gamma2</strong></td>
<td>Pdf</td>
<td>shape</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>rate</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>InverseGamma1</strong></td>
<td>Pdf</td>
<td>shape</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>scale</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>NonParametric</strong></td>
<td>Pdf</td>
<td>bins</td>
<td>Vector</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>probability</td>
<td>Vector</td>
<td></td>
</tr>
<tr>
<td><strong>MultiNonParametric</strong></td>
<td>vector</td>
<td>bins</td>
<td>Matrix</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pmf</td>
<td>probability</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td><strong>Empirical</strong></td>
<td>Pdf</td>
<td>data</td>
<td>Vector</td>
<td></td>
</tr>
<tr>
<td><strong>MultiEmpirical</strong></td>
<td>vector</td>
<td>data</td>
<td>Matrix</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pdf</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MultivariateNormal1</strong></td>
<td>vector</td>
<td>mean</td>
<td>Vector</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pdf</td>
<td>covarianceMatrix</td>
<td>Matrix</td>
<td></td>
</tr>
<tr>
<td><strong>MultivariateNormal2</strong></td>
<td>vector</td>
<td>mean</td>
<td>Vector</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pdf</td>
<td>precisionMatrix</td>
<td>Matrix</td>
<td></td>
</tr>
<tr>
<td><strong>MultivariateStudentT1</strong></td>
<td>vector</td>
<td>mean</td>
<td>Vector</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pdf</td>
<td>covarianceMatrix</td>
<td>Matrix</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>degreesOfFreedom</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td>Name</td>
<td>Return Type</td>
<td>Argument name</td>
<td>Argument Types</td>
<td>Comment</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>-------------</td>
<td>------------------------</td>
<td>----------------</td>
<td>---------</td>
</tr>
<tr>
<td>MultivariateStudentT2</td>
<td>vector Pdf</td>
<td>mean, precisionMatrix, degreesOfFreedom</td>
<td>Vector, Matrix, Real</td>
<td></td>
</tr>
<tr>
<td>NegativeBinomial2</td>
<td>Pdf</td>
<td>rate, overdispersion</td>
<td>Real, Real</td>
<td></td>
</tr>
<tr>
<td>StudentT1</td>
<td>Pdf</td>
<td>degreesOfFreedom</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td>StudentT2</td>
<td>Pdf</td>
<td>mean, scale, degreesOfFreedom</td>
<td>Real, Real, Real</td>
<td></td>
</tr>
<tr>
<td>Uniform1</td>
<td>Pdf</td>
<td>minimum, maximum</td>
<td>Real, Real</td>
<td></td>
</tr>
<tr>
<td>Wishart1</td>
<td>Matrix Pdf</td>
<td>scaleMatrix, degreesOfFreedom</td>
<td>Matrix, Real</td>
<td></td>
</tr>
<tr>
<td>Wishart2</td>
<td>Matrix Pdf</td>
<td>inverseScaleMatrix, degreesOfFreedom</td>
<td>Matrix, Real</td>
<td></td>
</tr>
<tr>
<td>InverseWishart1</td>
<td>Matrix Pdf</td>
<td>scaleMatrix, degreesOfFreedom</td>
<td>Matrix, Real</td>
<td></td>
</tr>
<tr>
<td>CategoricalNonordered1</td>
<td>Pmf</td>
<td>categoryProb</td>
<td>Vector</td>
<td></td>
</tr>
<tr>
<td>CategoricalOrdered1</td>
<td>Pmf</td>
<td>categoryProb</td>
<td>Vector</td>
<td></td>
</tr>
<tr>
<td>MixtureDistribution1</td>
<td>Pdf</td>
<td>weight, distributions</td>
<td>Vector, Vector of PDF</td>
<td></td>
</tr>
<tr>
<td>ZeroInflatedPoisson1</td>
<td>Pdf</td>
<td>rate, probabilityOfZero</td>
<td>Real, Real</td>
<td></td>
</tr>
</tbody>
</table>
10 Interoperability Guide

10.1 On interoperability

A key goal of the DDMoRe project is to have an interoperability framework in which models are written in a consistent language, translated to PharmML and from there converted to target software code. Before the DDMoRe project no existing language standard existed across target software used in pharmacometrics modelling, and while the underlying models could be expressed consistently in mathematical and statistical terms, the implementation of any given model varied by tool and by user according to their experience with a given target software tool.

There is some flexibility within MDL around how the user can express the mathematical and statistical models. Having flexibility allows the user to encode models quickly in a common language (MDL) which can then be shared with others and mutually understood. This flexibility also facilitates encoding in a given target when that language construct does not have a parallel in other tools. However, we STRONGLY encourage the user to encode the majority of models in a way that will facilitate interoperability. There are MDL constructs that facilitate interoperability - these generally appear as built-in functions which translate to specific constructs in PharmML and the target software. These constructs cover many typical models and are designed to allow the user to generate code quickly and have high confidence that it will be interoperable across tools.

The Model Description Language Interactive Development Environment (MDL-IDE) should assist the user in ensuring that the models encoded are valid MDL (and as a consequence, also valid PharmML). Not all models will result in code which can be readily converted to all target tools.

These interoperability constructs will be highlighted in the subsequent sections, but users should pay particular attention to sections on the use of GROUP_VARIABLES, INDIVIDUAL_VARIABLES and the MODEL_PREDICTION.

10.2 Dataset conventions

There are a number of conventions in preparing data for use in MDL and for target software.

- It is assumed that the SOURCE data file will be present as an ASCII comma-delimited text file (.csv).
- The data file should have a header row with names matching those in the DATA_INPUT_VARIABLES block.
- Data values should be numeric.
- Data columns with string or date:time values should have “use is ignore” for MDL.
- Null or missing values should be denoted by “.”.

Generally speaking, MDL follows NONMEM dataset conventions.

In addition the following restrictions should be observed for interoperability reasons:

- A column with “use is id” is mandatory. Values should be positive, non-zero integer, unique and contiguous.
- A column with “use is idv” is mandatory. Values should be positive, real. When the model is expressed using DEQ or COMPARTMENTS block the values must be monotonic increasing within ID. This constraint does not apply to analytical models. The first idv value is taken as the initial time for the model. The initial value does not need to be the same for all individuals, but it must not be lower than that of the first individual. date:time format is not supported for time.
- A column with “use is dv” is mandatory. This column can be any real value. This must have a null value for dosing records.
• When modelling multiple outcomes a column with “use is dvid” is required. Values should be positive, integer. Values should not be null for observation records.
• A column with “use is mdv” is optional. Valid values are 0 (observed), 1 (missing). When the observation is null or missing this column should have the value 1. It can take the value 1 when observations are present if this observation is to be ignored.
• A column with “use is evid” is optional. Valid values are 0 (observation record), 1 (dosing record), 4 (reset and dose record).
• A column with “use is amt” is optional. For dosing records this column must be have positive, real value.
• A column with “use is rate” is optional. This column must have positive, real values. This column can only be used in combination with a column with “use is amt”. If the value is zero then a bolus dose is assumed.
• A column with “use is addl” is optional. This column must have positive, real values. This column can only be used in combination with columns with “use is amt” and “use is ii”.
• A column with “use is ii” is optional. This column must have positive, real values. This column can only be used in combination with columns with “use is amt” and “use is addl” or “use is ss”.
• A column with “use is ss” is optional. Valid values are 0 (not at steady state), 1 (at steady state). This column can only be used in combination with columns with “use is amt” and “use is ii”.
• A column with “use is cmt” is optional unless there is more than one route of administration. This column must have positive, integer values. Values in this column should start at 1 and correspond to the order of ODEs specified in the \texttt{DEQ} block of the Model Object.
• Columns with “use is covariate”, “use is catCov” and “use is variable” are optional. These columns must not have positive, integer values. Values in this column should start at 1 and correspond to the order of ODEs specified in the \texttt{DEQ} block of the Model Object.

10.3 Multiple uses of dataset columns
Dataset columns cannot have multiple uses defined in \texttt{DATA_INPUT_VARIABLES}. The \texttt{DATA_DERIVED_VARIABLES} block can be used to specify additional uses for dataset variables. In the current MDL, scope for using \texttt{DATA_DERIVED_VARIABLES} is limited.

For example, if the user wants to specify different outcomes / observations conditional on a dataset variable like CMT, i.e. using CMT as DVID then they will need to create a dataset variable DVID mapping into CMT values appropriately.

10.4 Defining constants in the model
For interoperability, constant values in the model should be defined as \texttt{STUCTURAL_PARAMETERS} and fixed to a value in the Parameter Object.
For models expressed as systems of differential equations (DEQ block), model parameters can be set to constant values in the MODEL_PREDICTION block, but this may be inefficient in the target software translation.

In the current SEE, to ensure interoperability, model parameters should not be assigned a constant value in the GROUP_VARIABLES or INDIVIDUAL_VARIABLES block.

10.5 MODEL_PREDICTION block
To ensure Monolix interoperability, any variable used in the MODEL_PREDICTION block must be either:

- the independent variable
- defined in MODEL_PREDICTION
- declared in INDIVIDUAL_VARIABLES using {type is linear, ... }
- defined as “use is variable” in the DATA_INPUT_VARIABLES block of the Data Object

This implies in particular that STRUCTURAL_PARAMETERS, VARIABILITY_PARAMETERS, GROUP_VARIABLES and random variables defined in RANDOM_VARIABLES_DEFINITION cannot be used in MODEL_PREDICTION.

10.6 OBSERVATION block
For Monolix interoperability, different observations / outcomes must not share VARIABILITY_PARAMETERS and RANDOM_VARIABLE_DEFINITIONS.

For interoperability with Monolix, the residual error(s) $\sigma_{ij}^2$ must be Normal(0,1) random variables.
# 11 Glossary

## 11.1 Acronyms and Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMT</td>
<td>Dose amount</td>
</tr>
<tr>
<td>BOV</td>
<td>Between Occasion Variability (synonym for IOV - Inter-occasion variability)</td>
</tr>
<tr>
<td>COV</td>
<td>Covariance</td>
</tr>
<tr>
<td>CORR</td>
<td>Correlation</td>
</tr>
<tr>
<td>CTS</td>
<td>Clinical Trial Simulation</td>
</tr>
<tr>
<td>CWRES</td>
<td>Conditional Weighted Residual</td>
</tr>
<tr>
<td>DDMoRe</td>
<td>Drug Disease Model Resources</td>
</tr>
<tr>
<td>DoW</td>
<td>Description of Work</td>
</tr>
<tr>
<td>DV</td>
<td>Dependent Variable</td>
</tr>
<tr>
<td>EFPIA</td>
<td>European Federation of Pharmaceutical Industries and Associations</td>
</tr>
<tr>
<td>EMA</td>
<td>European Medicines Agency</td>
</tr>
<tr>
<td>EPS</td>
<td>Epsilon - Residual Unexplained Variability random effect</td>
</tr>
<tr>
<td>ETA</td>
<td>Empirical Bayes prediction of the inter-individual random effect in a PK or PD parameter</td>
</tr>
<tr>
<td>FDA</td>
<td>Food and Drug Administration</td>
</tr>
<tr>
<td>FIS</td>
<td>Framework Integration Service</td>
</tr>
<tr>
<td>ID</td>
<td>Individual</td>
</tr>
<tr>
<td>IDV</td>
<td>Independent variable</td>
</tr>
<tr>
<td>II</td>
<td>Inter-dose Interval</td>
</tr>
<tr>
<td>IMI</td>
<td>Innovative Medicines Initiative</td>
</tr>
<tr>
<td>IMI-JU</td>
<td>Innovative Medicines Initiative Joint Undertaking</td>
</tr>
<tr>
<td>IOF</td>
<td>Interoperability framework</td>
</tr>
<tr>
<td>IPRED</td>
<td>Individual Prediction</td>
</tr>
<tr>
<td>IWRES</td>
<td>Individual Weighted Residual</td>
</tr>
<tr>
<td>MDL</td>
<td>Model Description Language</td>
</tr>
<tr>
<td><strong>MDL-IDE</strong></td>
<td>Modelling Definition Language Integrated Development Environment</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>MDV</strong></td>
<td>Missing Dependent Variable</td>
</tr>
<tr>
<td><strong>MIF</strong></td>
<td>Mango Integration Framework</td>
</tr>
<tr>
<td><strong>MOG</strong></td>
<td>Modelling Object Group</td>
</tr>
<tr>
<td><strong>NONMEM</strong></td>
<td>NONLinear Mixed Effects Modelling (see Names)</td>
</tr>
<tr>
<td><strong>OBS</strong></td>
<td>Observed (value or data)</td>
</tr>
<tr>
<td><strong>PharmML</strong></td>
<td>Pharmacometrics Markup Language</td>
</tr>
<tr>
<td><strong>PD</strong></td>
<td>Pharmacodynamic</td>
</tr>
<tr>
<td><strong>PK</strong></td>
<td>Pharmacokinetic</td>
</tr>
<tr>
<td><strong>PK/PD</strong></td>
<td>Pharmacokinetic - Pharmacodynamic modelling</td>
</tr>
<tr>
<td><strong>PRED</strong></td>
<td>Population Prediction</td>
</tr>
<tr>
<td><strong>PROV-O</strong></td>
<td>Provenance Ontology</td>
</tr>
<tr>
<td><strong>PsN</strong></td>
<td>Perl Speaks NONMEM</td>
</tr>
<tr>
<td><strong>RDF</strong></td>
<td>Resource Description Framework</td>
</tr>
<tr>
<td><strong>RUV</strong></td>
<td>Residual Unexplained Variability</td>
</tr>
<tr>
<td><strong>sd</strong></td>
<td>Standard Deviation</td>
</tr>
<tr>
<td><strong>SE</strong></td>
<td>Standard Error</td>
</tr>
<tr>
<td><strong>SEE</strong></td>
<td>Stand-alone Execution Environment</td>
</tr>
<tr>
<td><strong>SO</strong></td>
<td>Standard Output</td>
</tr>
<tr>
<td><strong>TEL</strong></td>
<td>Task Execution Language. The working name within the DDMoRe project for the ddmore R package used to perform tasks with MDL and define pharmacometric workflow.</td>
</tr>
<tr>
<td><strong>TES</strong></td>
<td>Task Execution Server</td>
</tr>
<tr>
<td><strong>var</strong></td>
<td>Variance</td>
</tr>
<tr>
<td><strong>VPC</strong></td>
<td>Visual Predictive Check</td>
</tr>
<tr>
<td><strong>WP</strong></td>
<td>Work Package</td>
</tr>
<tr>
<td><strong>WRES</strong></td>
<td>Weighted Residual</td>
</tr>
<tr>
<td><strong>XML</strong></td>
<td>Extensible Markup Language</td>
</tr>
</tbody>
</table>
## 11.2 Definitions and System Names

<table>
<thead>
<tr>
<th>System Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annotation</td>
<td>A description attached to a model or element of a model; see RDF triple.</td>
</tr>
<tr>
<td>Bootstrap</td>
<td>Bootstrap is a tool for calculating bias, standard errors and confidence intervals of parameter estimates. It does so by generating a set of new datasets by sampling individuals with replacement from the original dataset, and fitting the model to each new dataset</td>
</tr>
<tr>
<td>Connector</td>
<td>A piece of software which enables modelling software to communicate with the interoperability framework</td>
</tr>
<tr>
<td>Converter</td>
<td>A piece of software which enables translation across languages (e.g. mdl to pharmML</td>
</tr>
<tr>
<td>End-User</td>
<td>A specific user role with privileges to manage only his/her own jobs queues, etc.</td>
</tr>
<tr>
<td>Extensible Markup Language (XML)</td>
<td>An open standard for exchanging structured documents and data over the internet that was introduced by the World Wide Web Consortium (W3C).</td>
</tr>
<tr>
<td>Metadata</td>
<td>Metadata (metacontent) is defined as data providing information about one or more aspects of the data, such as:</td>
</tr>
<tr>
<td></td>
<td>▪ Means of creation of the data</td>
</tr>
<tr>
<td></td>
<td>▪ Purpose of the data</td>
</tr>
<tr>
<td></td>
<td>▪ Time and date of creation</td>
</tr>
<tr>
<td></td>
<td>▪ Creator or author of data</td>
</tr>
<tr>
<td></td>
<td>▪ Placement on a computer network where the data was created</td>
</tr>
<tr>
<td></td>
<td>▪ Standards used</td>
</tr>
<tr>
<td>Monolix</td>
<td>A software for the analysis of nonlinear mixed effects models</td>
</tr>
<tr>
<td>MDL-IDE</td>
<td>Graphical user interface of the Interoperability Framework. It provides a framework within which files containing MDL code can be created and edited and Modelling &amp; Simulation workflows can be created and executed.</td>
</tr>
<tr>
<td>MDL</td>
<td>MDL is the Model Description Language (formerly MCL - Model Coding Language) the human writable and human readable language designed to describe pharmacometric models.</td>
</tr>
<tr>
<td>MLXTRAN</td>
<td>The language used to define models that are executed with Monolix.</td>
</tr>
<tr>
<td>NM-TRAN</td>
<td>The language used to define models that are executed with NONMEM.</td>
</tr>
<tr>
<td>NONMEM</td>
<td>A software for the analysis of nonlinear mixed effects models.</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.globomax.com/nonmem.htm">http://www.globomax.com/nonmem.htm</a></td>
</tr>
<tr>
<td>Ontology</td>
<td>An organization of some knowledge domain that is hierarchical and contains all</td>
</tr>
</tbody>
</table>
the relevant entities and their relations.
An ontology is used to define the relationships and objects that are used to define the RDF Triples that describe the data, models and results with a Pharmacometrics Workflow

| R | R is a free software environment for statistical computing and graphics. https://www.r-project.org/ |
| RDF Triple | An RDF Triple is a statement which relates one object to another. It is composed of three parts:
- the subject - the entity we are describing
- a predication - the relationship
- the object - the description
DDMoRe uses RDF triples to describe models held within the repository, for example:
“MODEL-000034765” “has author” “Lena Friberg” |

| Task Execution Service (TES) | Performs job-management within the Interoperability Framework. |
| WinBUGS | Windows implementation of the BUGS (Bayesian Inference Using Gibbs Sampling) project, concerned with flexible software for the Bayesian analysis of complex statistical models using Markov chain Monte Carlo (MCMC) methods. http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/contents.shtml |
| Pharmacometric Workflow | Tracking the evolution of a model and associated inferences from initial model to final model, capturing metadata and annotations that will facilitate creation of a run record, audit log, QC and reproducibility of all steps within the workflow. Each step in the Pharmacometric workflow may consist of a Task Workflow which defines the procedural steps required to perform a sequence of tasks for a given model. http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/contents.shtml |
| Task Workflow | A sequence of tasks and procedural steps which can be captured in a scriptable language, like R, facilitating reproducibility of the outputs for a given set of inputs. |
| Interoperability | One stated of the DDMoRe project is to provide the capability to define the model once and then use it across a variety of target software tools. We call this “Interoperability”. |
| Repository | Another stated aim of the DDMoRe project is to provide a library platform for pre-competitive sharing of models - disease models, drug models etc. We call this library the “Repository”. |
| PharmML | XML based exchange format for encoding of non-linear mixed effect models, trial design and modelling steps used in pharmacometrics. URL: pharmml.org |
| Standard Output | Tool-independent exchange format intended for storage of results in standardised |
form, enabling effective data exchange within complex workflows as well as to support the user in assessing, reviewing and reporting a modelling step.